ANALOG FILTER DESIGN

FEG2D3

-INW-

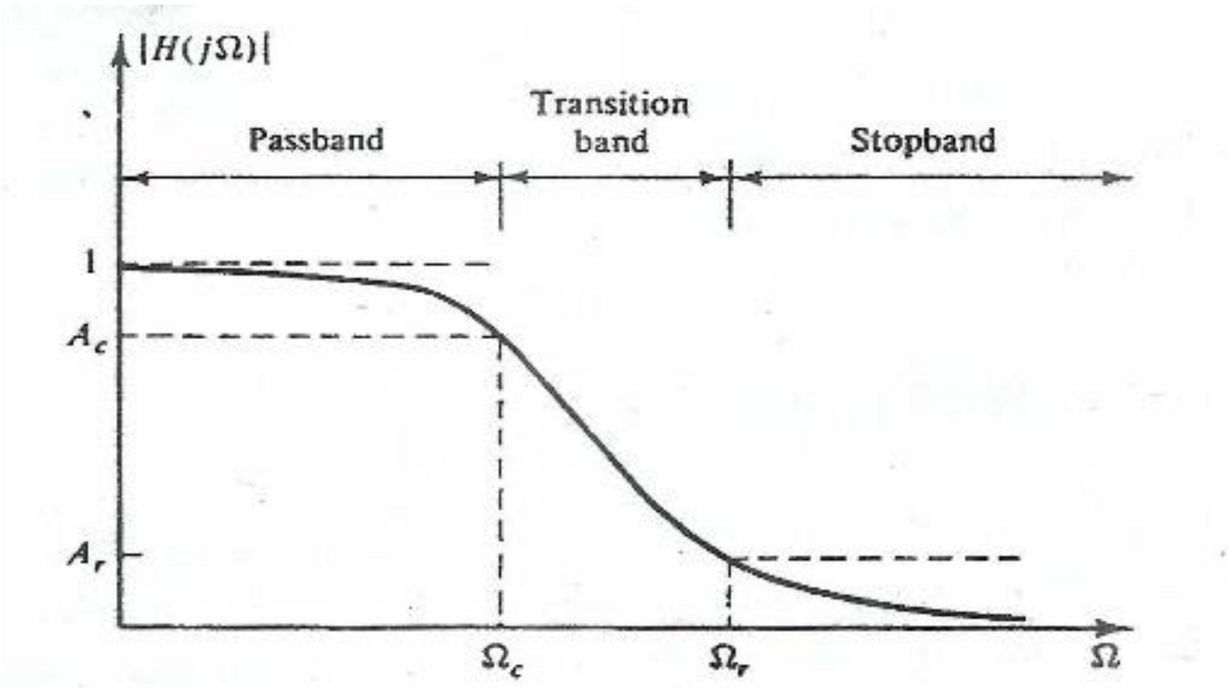
2016

PENDAHULUAN

- Di dalam perancangan filter-filter digital respons impuls tak terbatas diperlukan transformasi ke filter analog
- Diperlukan adanya pengetahuan filter analog yang dapat bertindak sebagai prototype untuk mendapatkan fungsi transfer filter H(s) yang dikehendaki
- Fungsi transfer filter H(s) diperoleh dari respons frekuensi filter $H(j\Omega)$ yang dirancang

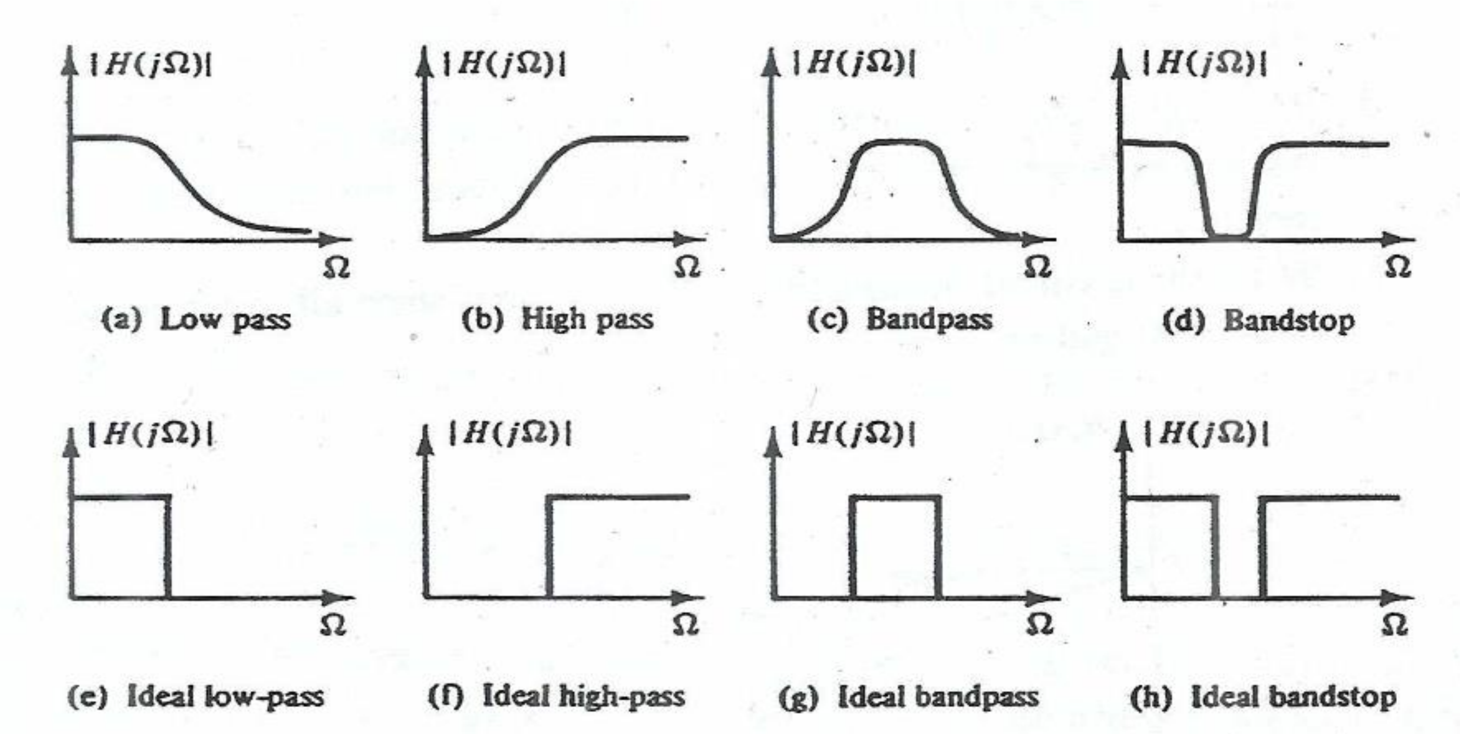
PENDAHULUAN

- Desain filter lowpas
- Respon frekuensi yang dibutuhkan



PENDAHULUAN

- Respon frekuensi



FILTER BUTTERWORTH

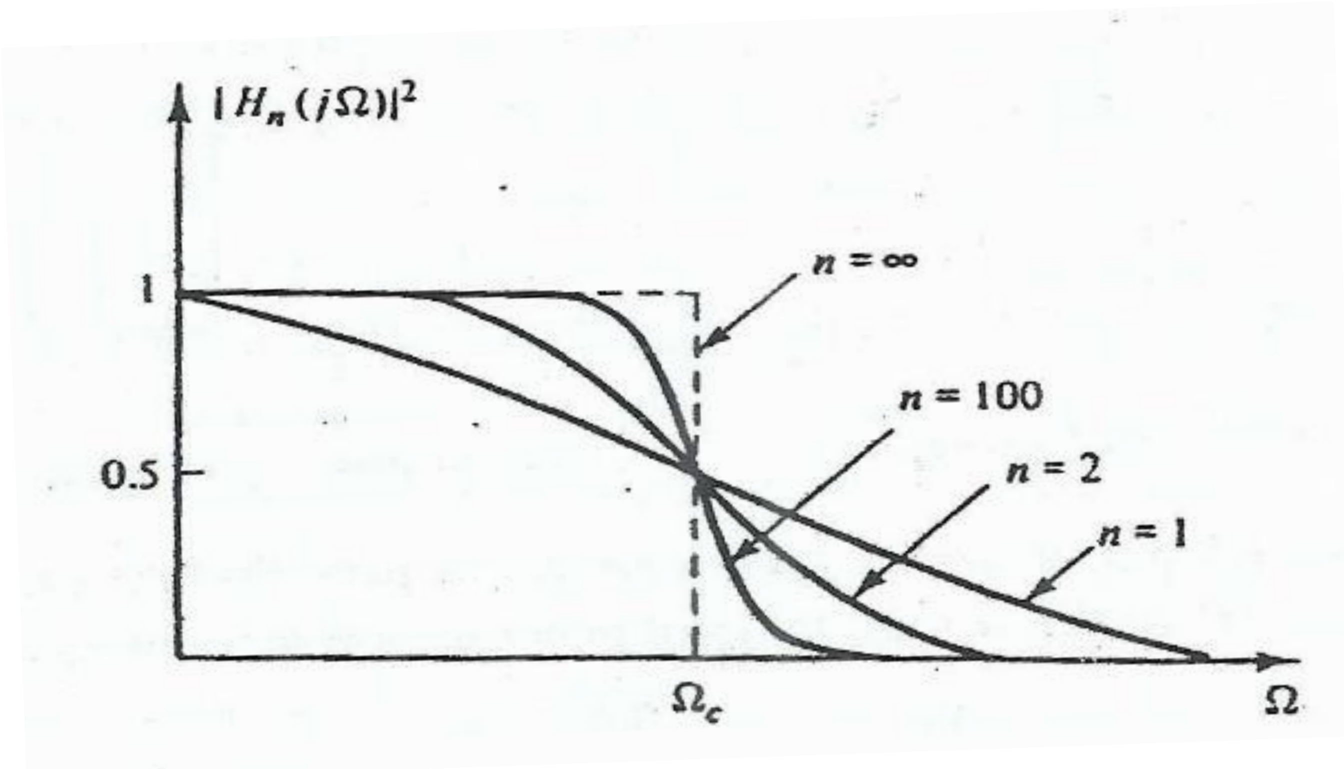
- Magnitude squared of frequency respons

$$|H_n(j\Omega)|^2 = \frac{1}{\left[1 + (\frac{\Omega}{\Omega_c})^{2n}\right]}$$

- $|H_n(j\Omega)|^2|_{\Omega=0} = 1$ untuk semua n
- $|H_n(j\Omega)|^2|_{\Omega=\Omega_c} = 0{,}5$ untuk semua n terbatas
- $|H_n(j\Omega)||_{\Omega=\Omega_c} = 0{,}7071$ dan
- $20 \log|H_n(j\Omega)||_{\Omega=\Omega_c} = -3{,}0103$ dB
- $|H_n(j\Omega)|^2$ adalah fungsi monoton menurun dengan varibel Ω
- Bila $n \to \infty$, $|H_n(j\Omega)|^2$ mendekati respon frekuensi LP ideal
- $|H_n(j\Omega)|^2$ disebut "maximally flat" di $\Omega = 0$

MAGNITUDE SQUARED FREQ RESPONSE

- Magnitude squared frequency response of the Butterworth filter for several different values of n



FILTER BUTTERWORTH

FILTER BUTTERWORTH

- Gain:

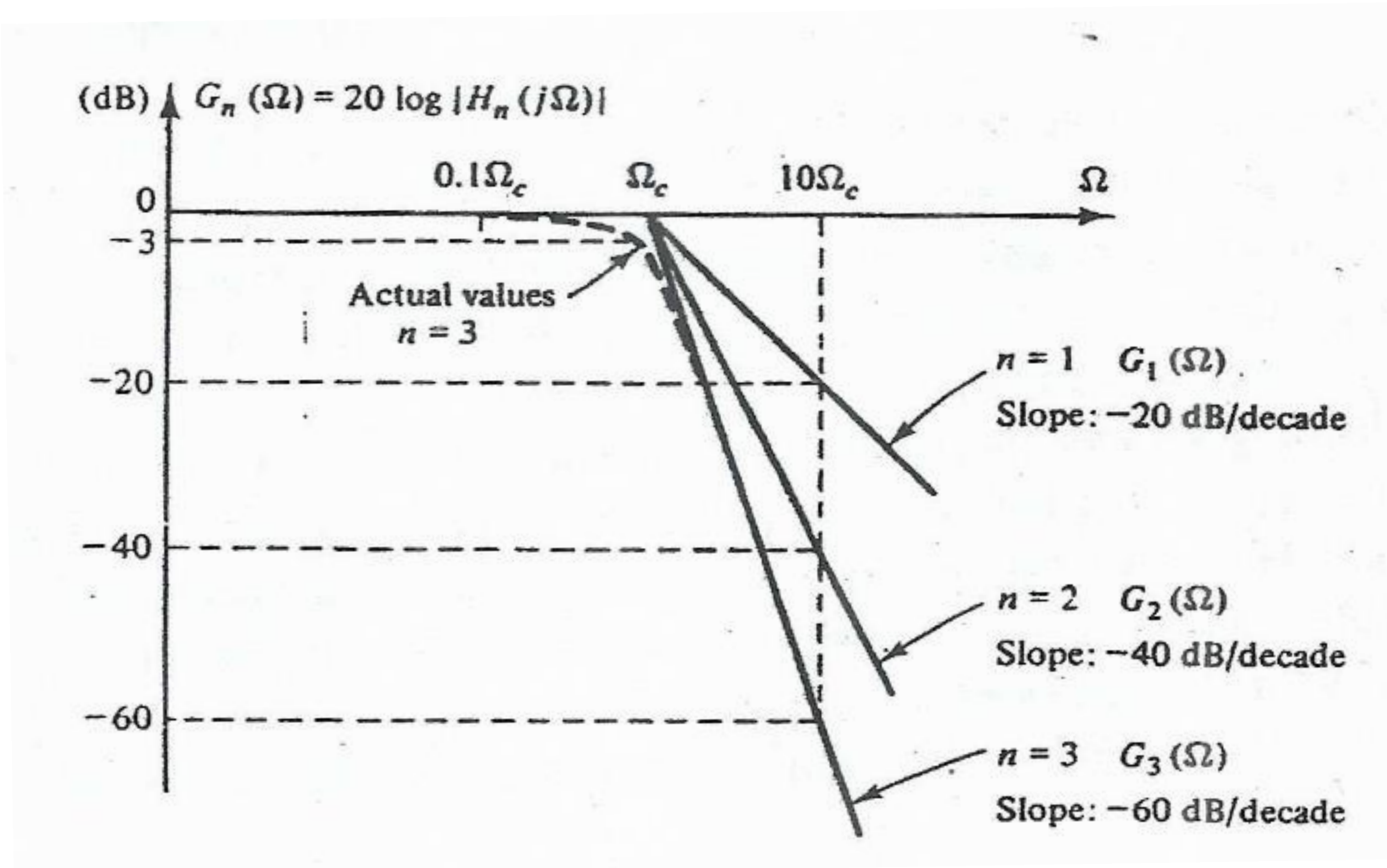
$$G_n(\Omega) = 20\log|H_n(j\Omega)| = 10\log|H_n(j\Omega)|^2$$

$$G_n(\Omega) = 10\log\left[\frac{1}{\left[1+(\frac{\Omega}{\Omega_c})^{2n}\right]}\right] = -10\log\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}\right]$$

Fungsi n

- **Untuk** $\Omega \ll \Omega_c$ → $G_n(\Omega) \approx 0\ \text{dB}$
- **Untuk** $\Omega \gg \Omega_c$ → $G_n(\Omega) \approx -20\text{n}\log\left|\frac{\Omega}{\Omega_c}\right|$

FILTER GAIN PLOT



FILTER BUTTERWORTH

- Filter Butterworth Low Pass ternormalisasi $\Omega_c = 1$ rad/detik
- Magnitude square respon frekuensi:

$$|H_n(j\Omega)|^2 = \frac{1}{1 + (\Omega)^{2n}}$$

- Fungsi transfer Filter Butterworth Low Pass ternormalisasi: H(s)
- Umumnya $s = \sigma + j\Omega$, bila $s = j\Omega$ → $\Omega = \frac{s}{j}$

$$|H_n(j\Omega)|^2 = H_n(j\Omega)H_n(-j\Omega) = \frac{1}{1 + (\Omega)^{2n}}$$

$$H_n(s)H_n(-s) = \frac{1}{1 + \left(\frac{s}{j}\right)^{2n}}$$

- Pole-pole dari $H_n(s)H_n(-s)$ dihitung dari akar penyebut:

$$1 + \left(\frac{s}{j}\right)^{2n} \text{ atau } s^{2n} = -1(j)^{2n} = (-1)^{n+1}$$

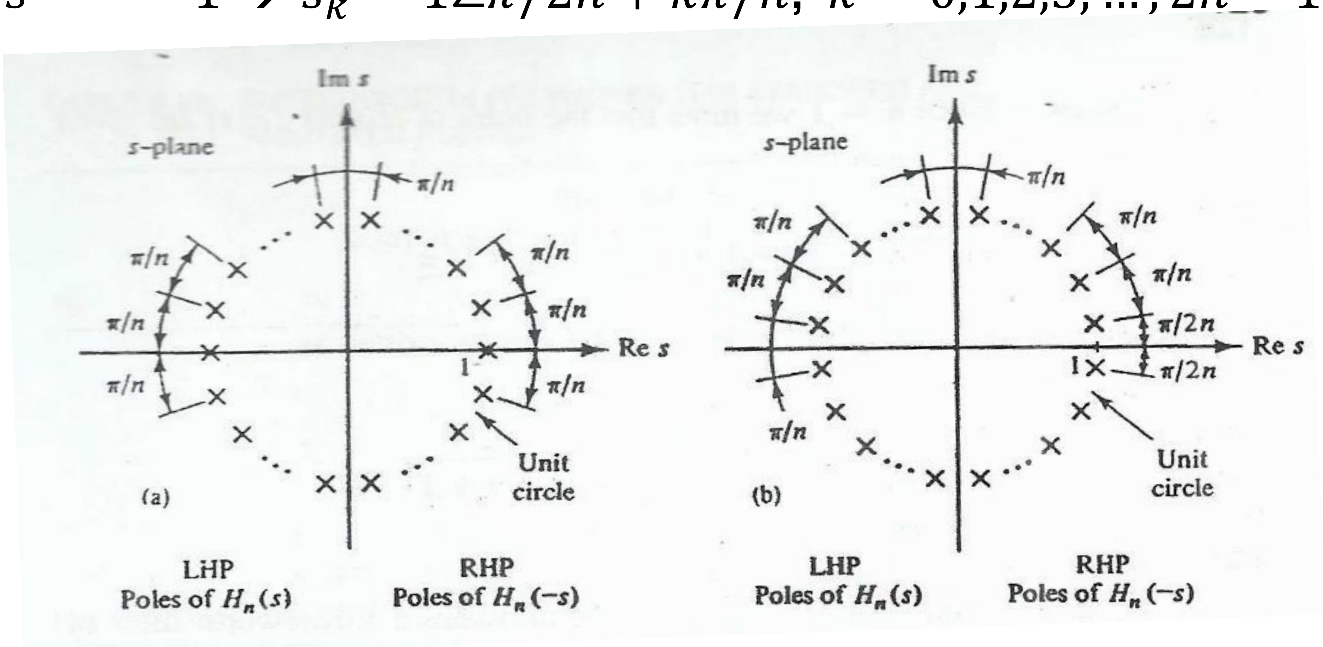
FILTER BUTTERWORTH

- Bila n ganjil:

$$s^{2n} = 1 \rightarrow s_k = 1\angle k\pi/n,\ k = 0,1,2,3,\ldots,2n-1$$

- Bila n genap

$$s^{2n} = -1 \rightarrow s_k = 1\angle \pi/2n + k\pi/n,\ k = 0,1,2,3,\ldots,2n-1$$



FILTER BUTTERWORTH

- Fungsi tranfser filter H(s) harus stabil dan kausal, maka pole-pole $H_n(s)$ harus berada di sebelah kiri sumbu $j\Omega$ pada bidang s

$$H_n(s) = \frac{1}{\prod_{pole\ seb\ kiri}(s - s_k)} = \frac{1}{B_n(s)}$$

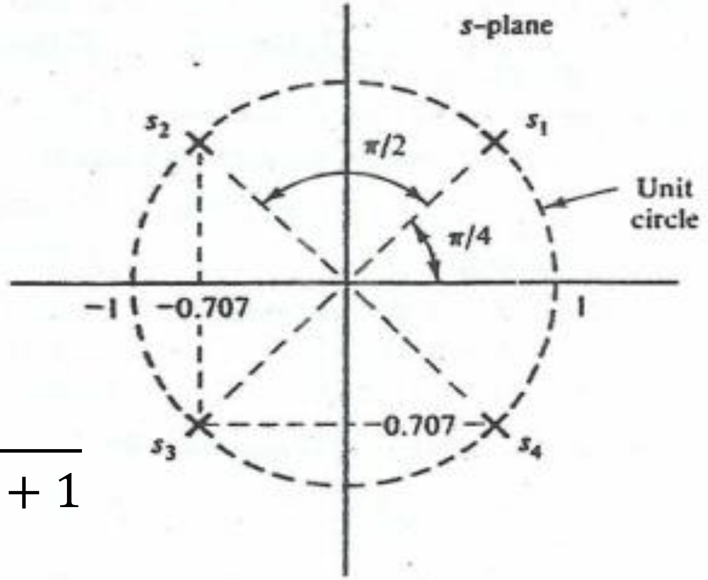
- Dimana s_k adalah pole pole $H_n(s)H_n(-s)$ yang berada di sebelah sumbu $j\Omega$ pada bidang s
- Polinomial $B_n(s)$ adalah polinomial Butterworth orde n

FILTER BUTTERWORTH

- **Fungsi transfer filter Butterworth ternormalisasi orde 1**
  - Pole: $s_1 = 1\angle 0 = 1, s_2 = 1\angle\pi = -1,$

$$H_1(s) = \frac{1}{s-(-1)} = \frac{1}{s+1}$$

- **Fungsi transfer filter Butterworth ternormalisasi orde 2**
  - Pole: $s_k = 1\angle\pi/2n + k\pi/n,\ k = 0{,}1{,}2{,}3$
  - $s_1 = 1\angle\frac{\pi}{4} = 0{,}707 + j0{,}707$
  - $s_2 = 1\angle\frac{3\pi}{4} = -0{,}707 + j0{,}707$
  - $s_3 = 1\angle\frac{5\pi}{4} = -0{,}707 - j0{,}707$
  - $s_4 = 1\angle\frac{7\pi}{4} = 0{,}707 - j0{,}707$



- Fungsi transfer:

$$H_2(s) = \frac{1}{(s-s_2)(s-s_3)} = \frac{1}{s^2+\sqrt{2}s+1}$$

FILTER BUTTERWORTH

- Fungsi transfer Filter Butterworth ternormalisasi $\Omega_c = 1$ rad/det

Orde Filter	$H_n(s) = \dfrac{1}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1} = \dfrac{1}{B_n(s)}$
n	Polynomial $B_n(s)$
1	$s + 1$
2	$s^2 + \sqrt{2s} + 1$
3	$s^3 + 2s^2 + 2s + 1$
4	$s^4 + 2{,}613s^3 + 3{,}414s^2 + 2{,}613s + 1$
5	$s^5 + 3{,}236s^4 + 5{,}236s^3 + 5{,}236s^2 + 3{,}236s + 1$
6	$s^6 + 3{,}863s^5 + 7{,}464s^4 + 9{,}141s^3 + 7{,}464s^2 + 3{,}863s + 1$
7	$s^7 + 4{,}494s^6 + 10{,}103s^5 + 14{,}606s^4 + 14{,}606s^3 + 10{,}103s^2 + 4{,}494s + 1$

Denominator coefficients for polynomials of the form $S_n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots\ldots\ldots\ldots + a_1 s + a_0$

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	1									
2	1	1.414								
3	1	2.000	2.000							
4	1	2.613	3.414	2.613						
5	1	3.236	5.236	5.236	3.236					
6	1	3.864	7.464	9.142	7.464	3.864				
7	1	4.494	10.098	14.592	14.592	10.098	4.494			
8	1	5.126	13.137	21.846	25.688	21.846	13.137	5.126		
9	1	5.759	16.582	31.163	41.986	41.986	31.163	16.582	5.759	
10	1	6.392	20.432	42.802	64.882	74.233	64.882	42.802	20.432	6.392

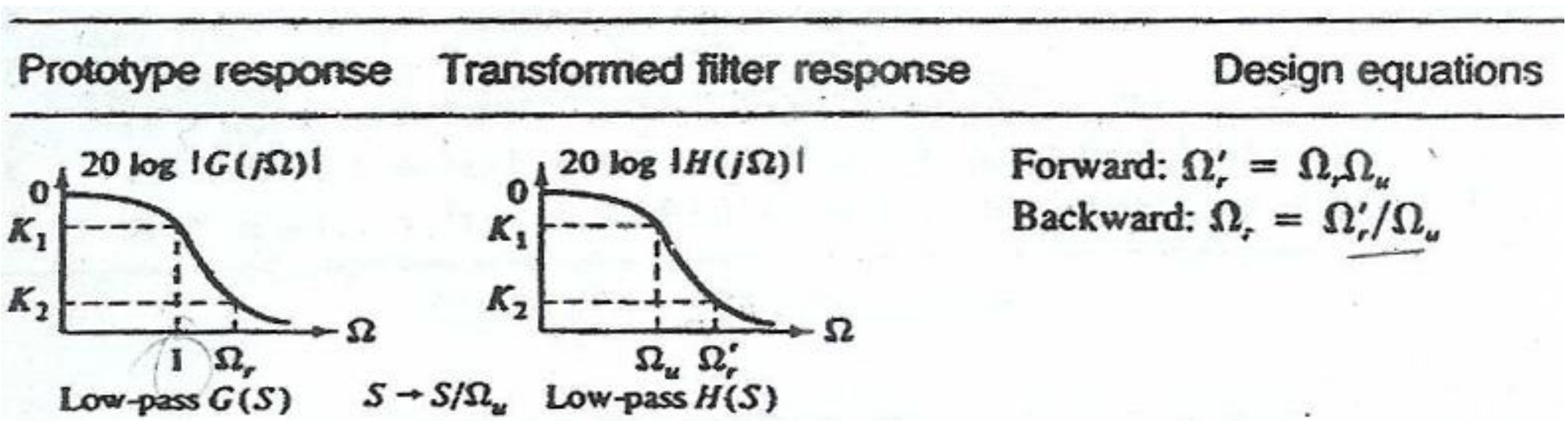
TRANSFORMASI ANALOG KE ANALOG

- Dalam praktek umumnya diinginkan membuat filter low-pass dengan $\Omega_c = \Omega_u \neq 1$ rad/detik
- Harus ditentukan $H(s)$ filter berdasarkan $H_n(s)$ LPF yang dipakai
- Bila s di $H_n(s)$ digatni dengan $\frac{s}{\Omega_u}$ maka akan diperoleh

$$H(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_u}} = H_n\left(\frac{s}{\Omega_u}\right)$$

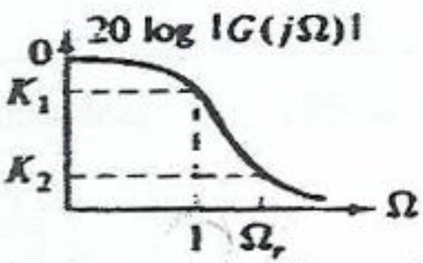
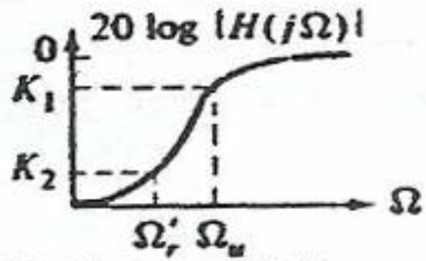
- Magnituda di $s = j\Omega$: $|H(j\Omega)| = \left|H(j\frac{\Omega}{\Omega_u})\right|$
- Magnituda di $s = j\Omega_u$: $|H(j\Omega_u)| = |H(j1)|$
- Artinya frekuensi cut-off $\Omega_c = 1\ rad/det$ pihdah $\Omega_c = \Omega_u$

TRANSFORMASI LPF KE LPF



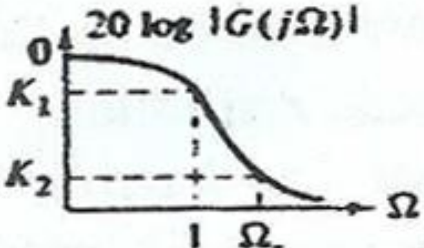
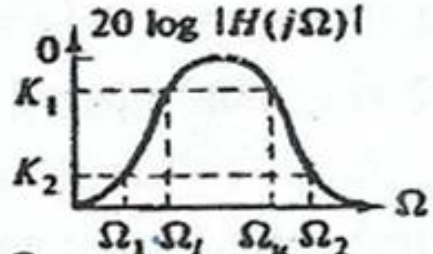
$$H_{1(s)} = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\frac{s}{\Omega_u}+1} = \frac{\Omega_u}{s+\Omega_u}$$

TRANSFORMASI LPF KE HPF

Prototype response	Transformed filter response	Design equations
 Low-pass $G(S)$	$S \to \Omega_u/S$  High-pass $H(S)$	Forward: $\Omega_r' = \Omega_u/\Omega_r$ Backward: $\Omega_r = \Omega_u/\Omega_r'$

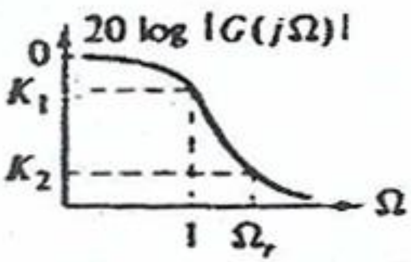
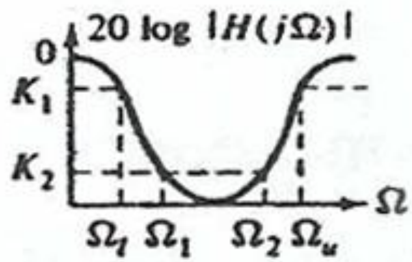
$$H_1(s) = \frac{1}{s+1} \to H(s) = \frac{1}{\frac{\Omega_u}{s}+1} = \frac{s}{s+\Omega_u}$$

TRANSFORMASI LPF KE BPF

Prototype response	Transformed filter response		Design equations
 Low-pass $G(S)$	$S \rightarrow \dfrac{S^2 + \Omega_l\Omega_u}{S(\Omega_u - \Omega_l)}$	 Bandpass $H(S)$	Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$ $\Omega_1 = (\Omega_r^2\Omega_{av}^2 + \Omega_l\Omega_u)^{1/2} - \Omega_{av}\Omega_r$ $\Omega_2 = (\Omega_r^2\Omega_{av}^2 + \Omega_l\Omega_u)^{1/2} + \Omega_{av}\Omega_r$ Backward: $\Omega_r = \min\{\|A\|, \|B\|\}$ $A = (-\Omega_1^2 + \Omega_l\Omega_u)/[\Omega_1(\Omega_u - \Omega_l)]$ $B = (+\Omega_2^2 - \Omega_l\Omega_u)/[\Omega_2(\Omega_u - \Omega_l)]$

$$H_1(s) = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\dfrac{s^2 + \Omega_l\Omega_u}{s(\Omega_u - \Omega_l)} + 1} = \frac{s(\Omega_u - \Omega_l)}{s^2 + s(\Omega_u - \Omega_l) + \Omega_l\Omega_u}$$

TRANSFORMASI LPF KE BSF

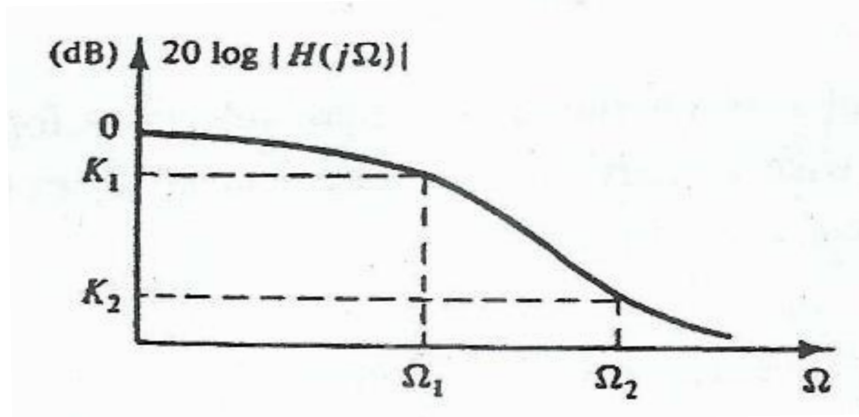
Prototype response	Transformed filter response	Design equations
 Low-pass $G(S)$	 Bandstop $H(S)$	Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$ $\Omega_1 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} - \Omega_{av}/\Omega_r$ $\Omega_2 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} + \Omega_{av}/\Omega_r$ Backward: $\Omega_r = \min\{\lvert A\rvert, \lvert B\rvert\}$ $A = \Omega_1(\Omega_u - \Omega_l)/[-\Omega_1^2 + \Omega_l\Omega_u]$ $B = \Omega_2(\Omega_u - \Omega_l)/[-\Omega_2^2 + \Omega_l\Omega_u]$

$$S \to \frac{S(\Omega_u - \Omega_l)}{S^2 + \Omega_l\Omega_u}$$

$$H_1(s) = \frac{1}{s+1} \to H(s) = \frac{1}{\frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l\Omega_u} + 1} = \frac{s^2 + \Omega_l\Omega_u}{s^2 + s(\Omega_u - \Omega_l) + \Omega_l\Omega_u}$$

DESAIN FILTER LP-BUTTERWORTH

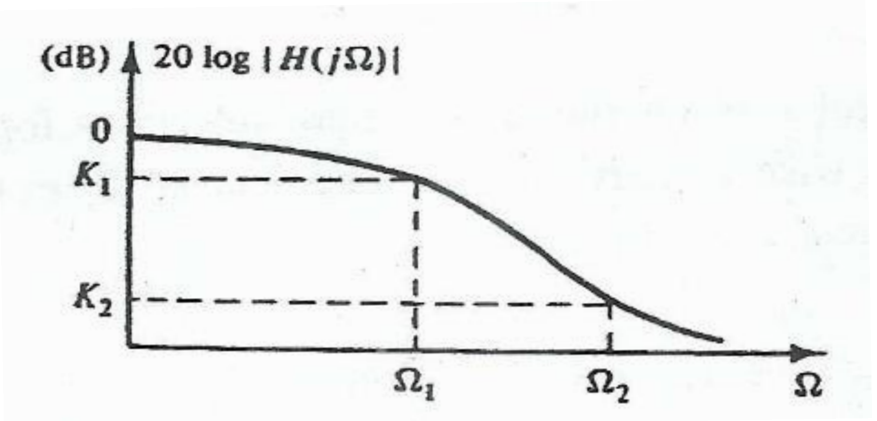
- Respon frekeunsi yang diinginkan



- Syarat:
- $0 \geq 20\log|H(j\Omega)| \geq K_1$, untuk semua $\Omega \leq \Omega_1$
- $20\log|H(j\Omega)| \leq K_2$, untuk semua $\Omega \geq \Omega_2$.
- Respon frekuensi filter LP Butterworth hanya ditentukan oleh n dan Ω_c
- Dari persamaan $|Hn(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$
- Maka $10log\left[\frac{1}{1+\left(\frac{\Omega_1}{\Omega_c}\right)^{2n}}\right] = K_1$, dan $10log\left[\frac{1}{1+\left(\frac{\Omega_2}{\Omega_c}\right)^{2n}}\right] = K_2$

DESAIN LP-BUTTERWORTH

- Respons frekuensi LPF yang diinginkan



- Diperoleh:

$$\left(\frac{\Omega_1}{\Omega_c}\right)^{2n} = 10^{-\frac{K_1}{10}} - 1 \; dan \left(\frac{\Omega_2}{\Omega_c}\right)^{2n} = 10^{-\frac{K_2}{10}} - 1$$

$$\left(\frac{\Omega_1}{\Omega_c}\right)^{2n} = \frac{10^{-\frac{K_1}{10}} - 1}{10^{-\frac{K_2}{10}} - 1} \quad \rightarrow \quad n = \left\lceil \frac{\log_{10}\left[\frac{10^{-\frac{K_1}{10}} - 1}{10^{-\frac{K_2}{10}} - 1}\right]}{2\log_{10}\left(\frac{\Omega_1}{\Omega_c}\right)} \right\rceil$$

DESAIN FILTER LP-BUTTERWORTH

- Nilai n yang diperoleh akan memberikan 2 nilai Ω_c yang berbeda:

$$\left(\frac{\Omega_1}{\Omega_c}\right)^{2n} = 10^{-\frac{K_1}{10}} - 1 \; dan \left(\frac{\Omega_2}{\Omega_c}\right)^{2n} = 10^{-\frac{K_2}{10}} - 1$$

- Bila diinginkan memenuhi persyaratan dengan tepat untuk nilai Ω_1 dan mendapat hasil lebih baik untuk Ω_2, dipakai:

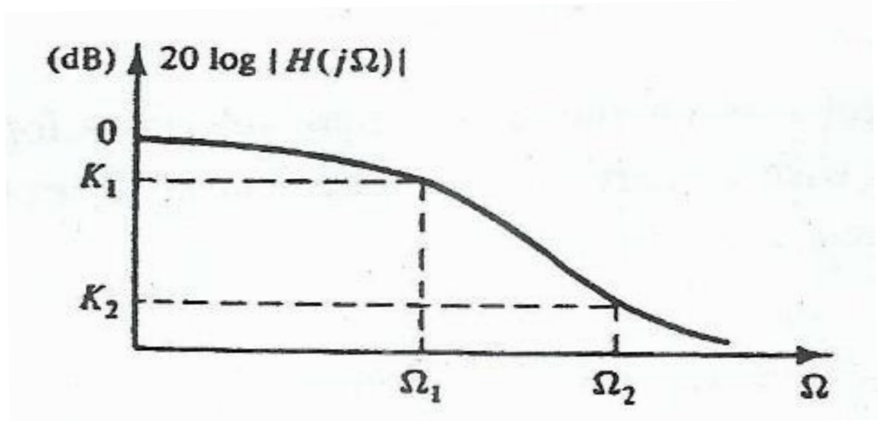
$$\Omega_{c1} = \frac{\Omega_1}{\left(10^{-\frac{K_1}{10}} - 1\right)^{\frac{1}{2n}}}$$

- Bila diinginkan memenuhi persyaratan dengan tepat untuk nilai Ω_2 dan mendapat hasil lebih baik untuk Ω_1, dipakai:

$$\Omega_{c2} = \frac{\Omega_2}{\left(10^{-\frac{K_2}{10}} - 1\right)^{\frac{1}{2n}}}$$

- Bisa dipilih $\Omega_{c1} < \Omega_c < \Omega_{c2}$

EXAMPLE



- Rancang filter LP analog Butterworth yang memiliki gain -2dB atau lebih baik pada frekuensi 20 rad/detik serta mempunyai redaman paling sedikit 10dB pada frekuensi 30 rad/detik
- $\Omega_1 = 20, K_1 = -2; \qquad \Omega_2 = 30, K_2 = -10$
- $n = \left\lceil \frac{\log_{10}\left[\frac{10^{-\frac{K_1}{10}}-1}{10^{-\frac{K_2}{10}}-1}\right]}{2\log_{10}\left(\frac{\Omega_1}{\Omega_2}\right)} \right\rceil = 3{,}37 = 4$

$$\Omega_{c1} = \frac{\Omega_1}{\left(10^{-\frac{K_1}{10}} - 1\right)^{\frac{1}{2n}}} = \frac{20}{(10^{0,2} - 1)^{1/8}} = 21{,}3868$$

EXAMPLE

- Filter low-pass Butterworth ternormalisasi dengan

$\Omega_c = 1\, rad/det$ dan $n = 4$

$$H_4(s) = \frac{1}{s^4 + 2{,}613s^3 + 3{,}414s^2 + 2{,}613s + 1}$$

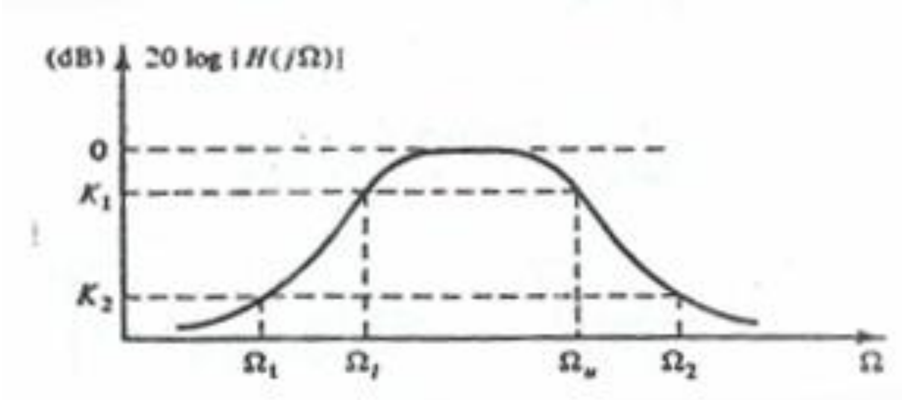
- Dengan transformasi LP ke LP, $s \rightarrow \frac{s}{\Omega_c}$,
- dengan $\Omega_c = 21{,}3868 rad/det$

$$H(s) = H_4(s)|_{s \rightarrow \frac{s}{21{,}3868}}$$

$$H(s) = \frac{1}{\left(\frac{s}{21{,}3868}\right)^4 + 2{,}613\left(\frac{s}{21{,}3868}\right)^3 + 3{,}414\left(\frac{s}{21{,}3868}\right)^2 + 2{,}613\left(\frac{s}{21{,}3868}\right) + 1}$$

DESAIN FILTER BP-BUTTERWORTH

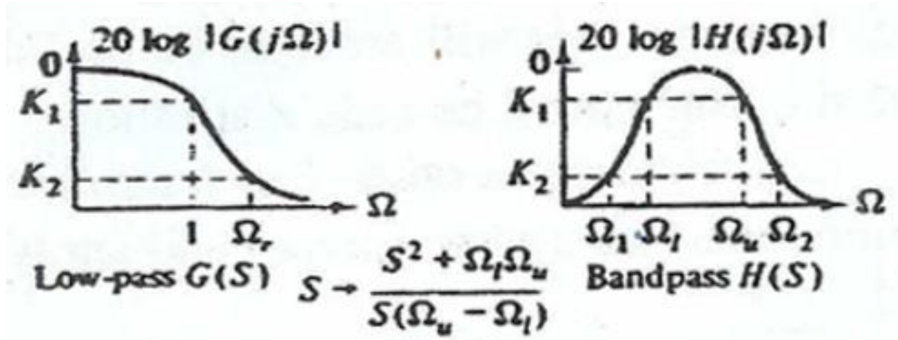
- Respons frekuensi yang diinginkan



- Persyaratan:
- $20 \log|H(j\Omega)| \leq K_2$, untuk semua $\Omega \leq \Omega_1$
- $0 \geq 20 \log|H(j\Omega)| \geq K_1$, untuk semua $\Omega_l \leq \Omega \leq \Omega_u$
- $20 \log|H(j\Omega)| \leq K_2$, untuk semua $\Omega \geq \Omega_2$

DESAIN FILTER BP-BUTTERWORTH

- Bila $H_{LP}(s)$ adalah LPF dengan $\Omega_c = 1 rad/detik$ dan frekuensi stop band adalah Ω_r



$$H_{BP}(s) = HLP(s)|_{s \to \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}}$$

$$\Omega_r = min\{|A|, |B|\}$$

$$A = \frac{-{\Omega_1}^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)} \text{ dan } B = \frac{{\Omega_2}^2+\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)}$$

EXAMPLE

- Rancang filter BP analog dengan spesifikasi:
  - Penguatan (gain)=$-3{,}0103dB$ pada $\Omega_l = 2\pi(50)rad/det$ dan $\Omega_u = 2\pi(20000)\ rad/det$
  - Redaman di stop band minimum $20dB$ pada $\Omega_1 = 2\pi(20)rad/det$ dan $\Omega_2 = 2\pi(50000)rad/det$
  - Respon frekuensi monotonic
- Jawab:
- Persyaratan respon frekuensi monotonic dipenuhi oleh filter Butterworth
- Frekuensi kritis:
- $\Omega_1 = 2\pi(20) = 125{,}663\ rad/det$
- $\Omega_l = 2\pi(50) = 314{,}159\ rad/det$
- $\Omega_u = 2\pi(20000) = 1{,}2566.10^5 rad/det$
- $\Omega_2 = 2\pi(50000) = 2{,}8274.10^5 rad/det$

EXAMPLE

- Filter LP prototype harus memenuhi:
- $0 \geq 20 \log|H_{LP(j1)}| \geq -3{,}0103\ dB$
- $20 \log|H_{LP(j\Omega_r)}| \leq -20\ dB$
- $A = \frac{-\Omega_1{}^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)} = 2{,}5053$
- $B = \frac{\Omega_2{}^2 + \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)} = 2{,}2545$

$\Omega_r = min\{|A|, |B|\}$

$\Omega_r = 2{,}2545$

- $n = \left\lceil \frac{\log_{10}\left[\frac{10^{0,301}-1}{10^{2}-1}\right]}{2\log_{10}(2{,}2545)} \right\rceil = \lceil 2{,}829 \rceil = 3$

$H_{LP}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$

- Transformasi LP → BP

$$s \to \frac{s^2 + \Omega_l\Omega_u}{s(\Omega_u - \Omega_l)} = \frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.105)}$$

EXAMPLE

$$H_{BP(s)} = \frac{1}{\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.105)}\right)^3 + 2\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.105)}\right)^2 + 2\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.105)}\right) + 1}$$

$$H_{BP}(s)$$

$$= \frac{1{,}9695.10^{15}s^3}{s^6 + 2{,}5069.10^5s^5 + 3{,}1543.10^{10}s^4 + 1{,}9893.10^{15}s^3 + 1{,}2452{,}10^{18}s^2 + 3{,}9072.10^{20}s + 6{,}1529.1022}$$

FILTER CHEBYSHEV

FILTER CHEBYSHEV

- Ada 2 type filter chebyshev:
  - Type 1: mempunyai ripple di pass band
  - Type 2: mempunyai ripple di stop band
- Filter low-pass Chebyshev type 1 ternormalisaasi mempunyai persamaan respons frekuensi magnitude squared:

$$|Hn(j\Omega)|^{2} = \frac{1}{1+\varepsilon^2 T_n{}^2(\Omega)} \qquad , n = 1,2,3, \ldots$$

- Dimana $T_n(\Omega)$ adalah polinomial Chebyshev orde n
- Polinomial Chebyshev dapat dihasilkan melalui rumus rekursif:

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \quad , n > 2$$

* Dimana $T_{0(x)} = 1 \; dan \; T_1(x) = x$

FILTER CHEBYSHEV

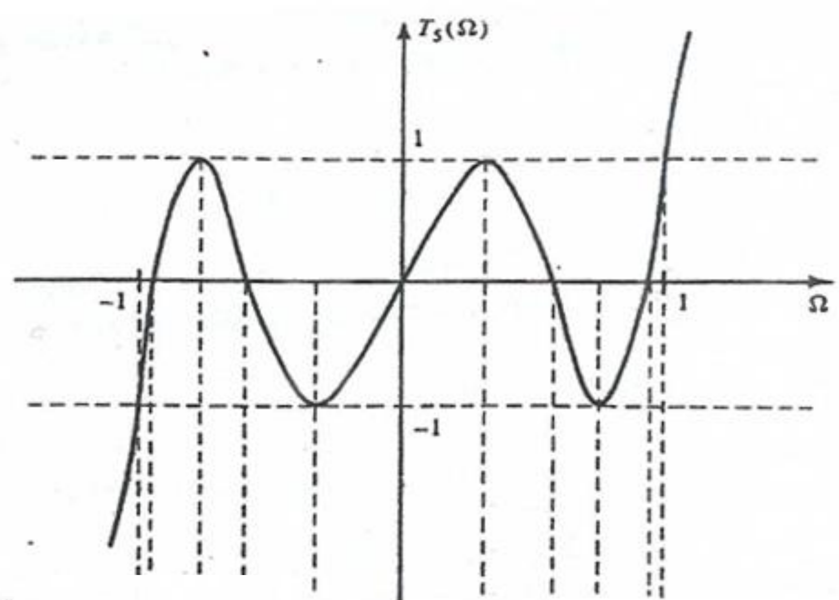
- Polinomial Chebyshev $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \quad , n > 2$

TABLE 3.3 THE FIRST TEN CHEBYSHEV POLYNOMIALS

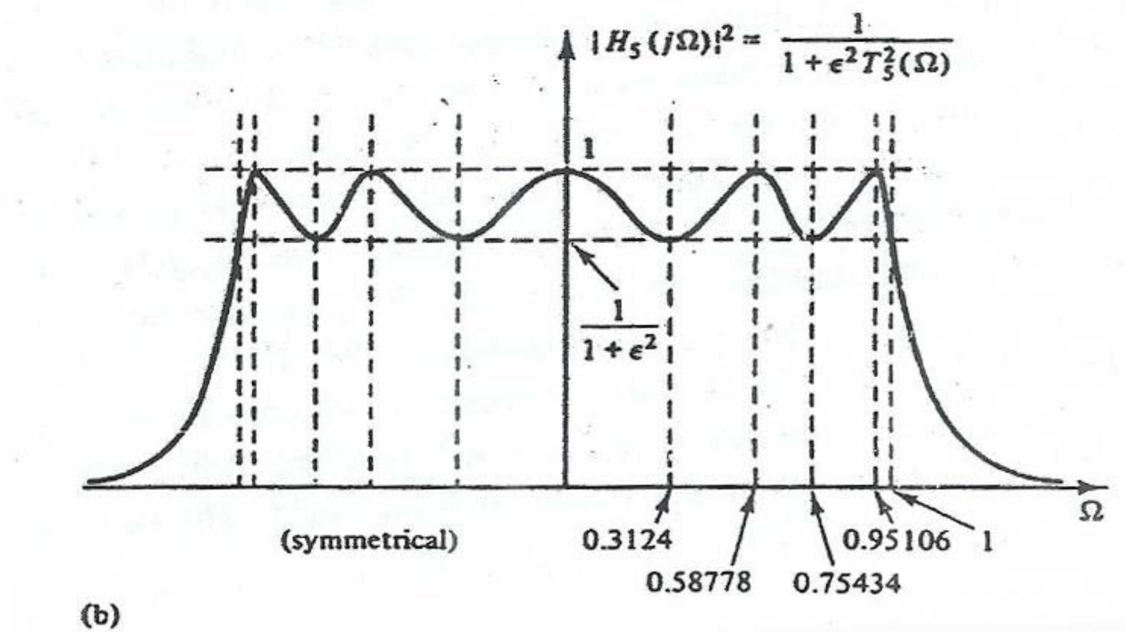
n	$T_n(x)$
0	1
1	x
2	$2x^2 - 1$
3	$4x^3 - 3x$
4	$8x^4 - 8x^2 + 1$
5	$16x^5 - 20x^3 + 5x$
6	$32x^6 - 48x^4 + 18x^2 - 1$
7	$64x^7 - 112x^5 + 56x^3 - 7x$
8	$128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$
9	$256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$
10	$512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1$

FILTER CHEBYSHEV

- Polinomial $T_5(x)$



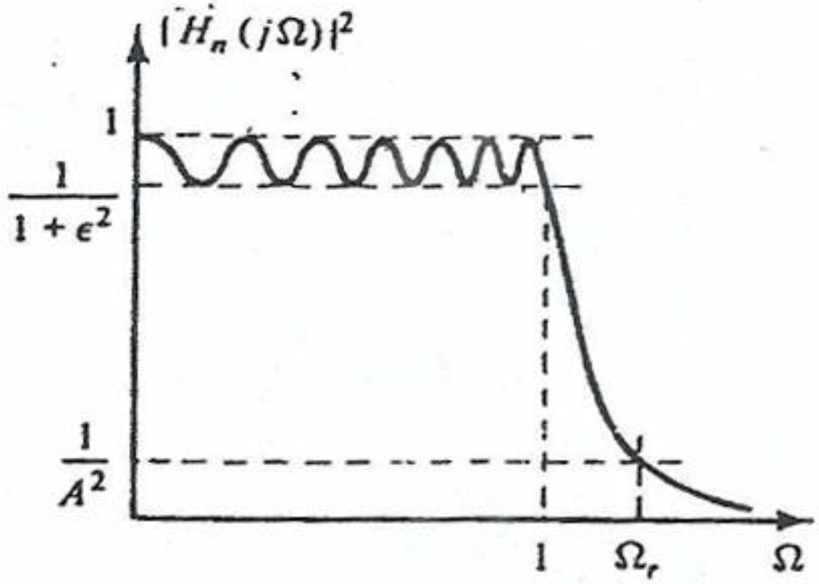
- $|H_5(j\Omega)|^2$ type 1



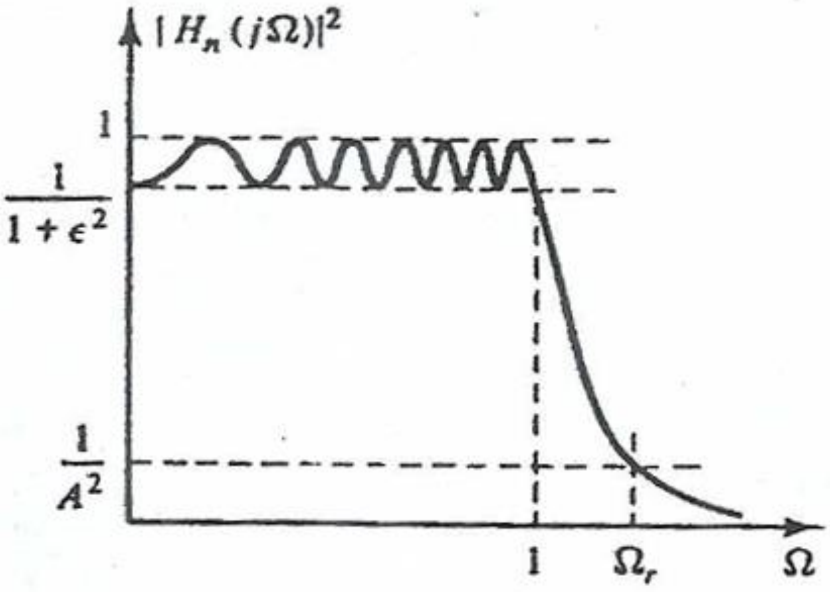
FILTER CHEBYSHEV

- n ganjil

n genap



n odd



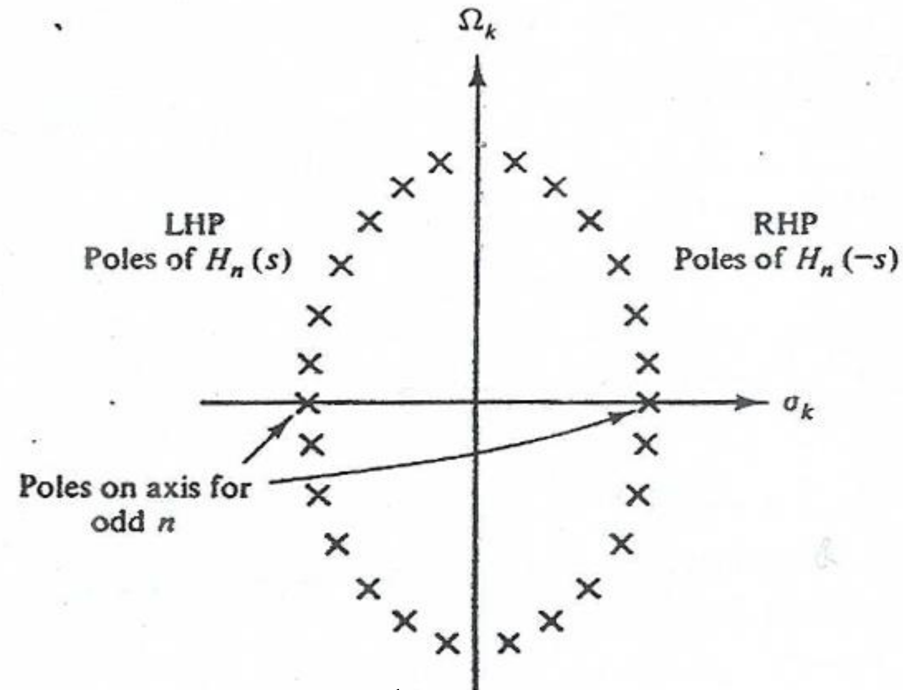
n even

FILTER CHEBYSHEV

- Sifat filter Chebyshev type 1
- Di daerah pass-band: $|Hn(j\Omega)|^{2}$ berosilasi antara 1 dan $\frac{1}{(1+\varepsilon^2)}$ disebut equiripple, dan pada frekuensi cutoff $\Omega = 1$ nilainya $\frac{1}{(1+\varepsilon^2)}$
- Di daerah band transisi dan stop-band, $|Hn(j\Omega)|^{2}$ nilainya monoton turun. Stop-band dimulai di Ω_r, dimana $|Hn(j\Omega)|^{2} = \frac{1}{A^2}$
- Fungsi transfer filter:
- $H(s)$ harus stabil dan kausal, maka pole-pole $H_n(s)$ harus berada di sebelah kiri sumbu $j\Omega$ pada bidang s
- Pole-pole dari $H_n(s)\ H_n(-s)$ dihitung dari akar penyebut:

$$1 + \varepsilon^2 T_n{}^2\left(\frac{s}{j}\right) = 0$$

FILTER CHEBYSHEV



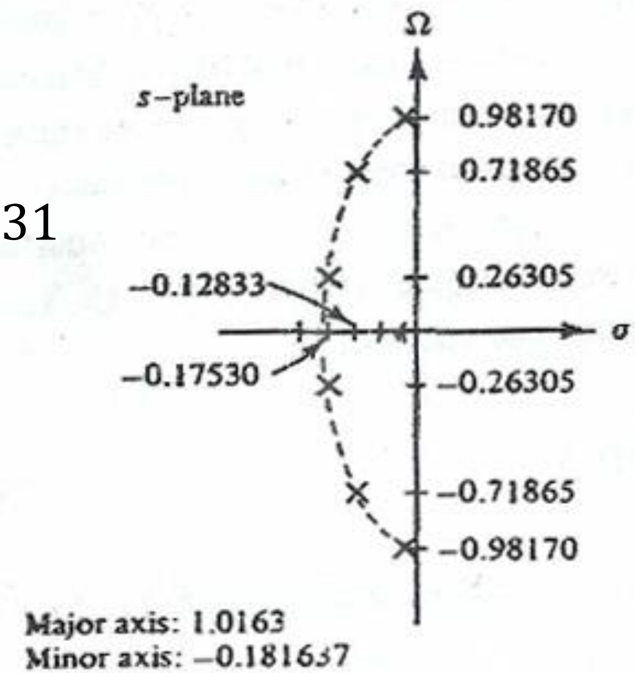
- Tempat kedudukan pole-pole
- $H_n(s)\ H_n(-s)$
- Bila pole $s_k = \sigma_k + j\Omega_k$
- Memenuhi persamaan
- $\frac{{\sigma_k}^2}{a^2} + \frac{{\Omega_k}^2}{b^2} = 1$
- Dimana

$$a = \frac{1}{2}\left\{\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right\}^{\frac{1}{n}} - \frac{1}{2}\left\{\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right\}^{-\frac{1}{n}}$$

$$b = \frac{1}{2}\left\{\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right\}^{\frac{1}{n}} + \frac{1}{2}\left\{\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right\}^{-\frac{1}{n}}$$

FILTER CHEBYSHEV

- $\sigma_k = -a\sin\left[\frac{(2k-1)\pi}{2n}\right] \quad , k = 1, \dots, 2n$
- $\Omega_k = b\cos\left[\frac{(2k-1)\pi}{2n}\right] \quad , k = 1, \dots, 2n$
- Pole H(s)
- $n = 6$
- $\varepsilon = 0{,}7647831$



$$\sigma_k = -a \sin\left[\frac{(2k-1)}{2n}\pi\right]$$

$$\Omega_k = b \cos\left[\frac{(2k-1)}{2n}\pi\right]$$

k	σ_k	Ω_k
1	−0.0469732	0.9817052
2	−0.1283332	0.7186581
3	−0.1753064	0.2630471
4	−0.1753064	−0.2630471
5	−0.1283332	−0.7186581
6	−0.0469732	−0.9817052

FILTER CHEBYSHEV

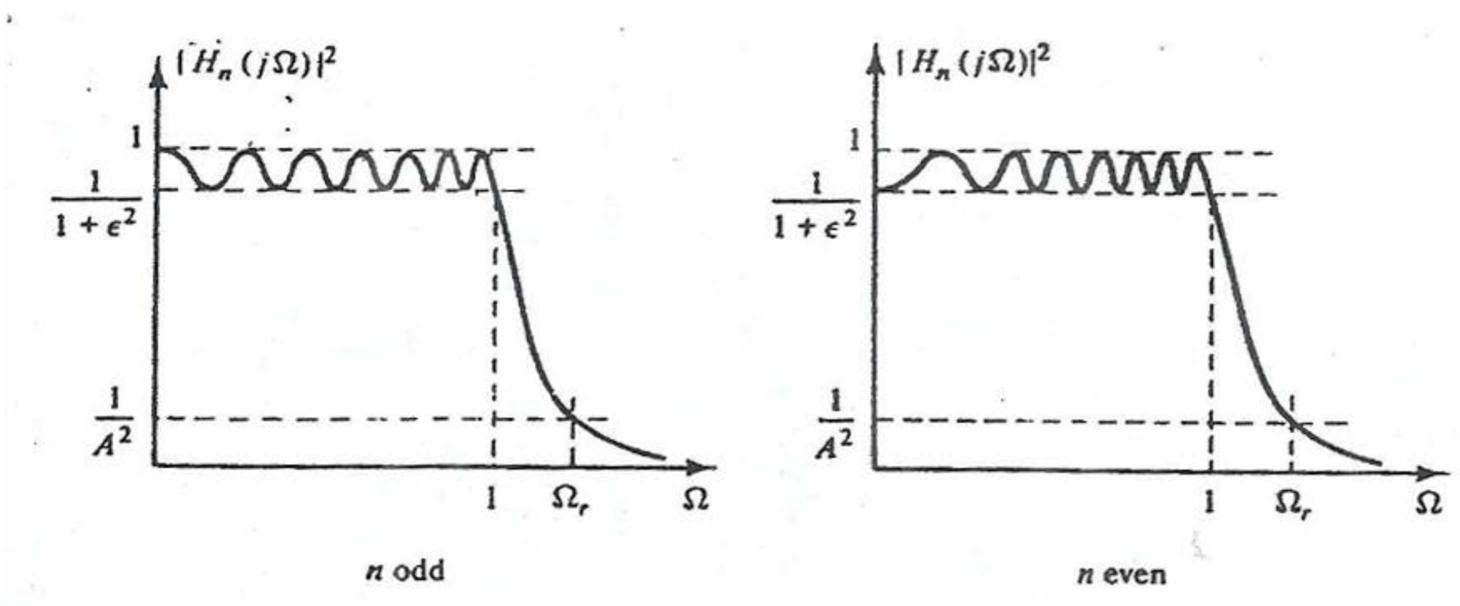
- Fungsi transfer filter $H(s)$ harus stabil dan kausal, maka pole-pole $H_{n(s)}$ harus berada di sebelah kiri sumbu $j\Omega$ pada bidang s
- $Hn(s) = \frac{K}{\Pi_{pole\ sebelah\ kiri}(s-s_k)} = \frac{K}{V_n(s)}$
- Dimana s_k adalah pole-pole $H_n(s)\ H_n(-s)$ yang berada di sebelah kiri sumbu $j\Omega$ pada bidang s
- K adalah faktor normalisasi, yang membuat nilai:

$$H(0) = \begin{cases} 1 & n\ ganjil \\ \frac{1}{\sqrt{1+\varepsilon^2}} & n\ genap \end{cases} \rightarrow K = \begin{cases} V_n(0) = b0 & n\ ganjil \\ \frac{V_n(0)}{\sqrt{1+\varepsilon^2}} & n\ genap \end{cases}$$

$$V_n(s) = sn + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$$

FILTER CHEBYSHEV

- Derajat filter $n = \left\lceil \frac{\log_{10}\left[g+\sqrt{g^2-1}\right]}{\log_{10}\left[\Omega_r+\sqrt{{\Omega_r}^2-1}\right]} \right\rceil$
- Dimana $A = \frac{1}{|H_n(j\Omega_r)|}$ dan $g = \sqrt{\frac{A^2-1}{\varepsilon^2}}$



- Chebyshev filters
- $|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2+T_n{}^2(\Omega)}$, $Hn_{(S)} = \frac{K_n}{V_n(s)}$,

$$K_n = \begin{cases} \frac{b_0}{\sqrt{1+\varepsilon^2}} & untuk\ n\ genap \\ b_0 & untuk\ n\ ganjil \end{cases}$$

$$V_n(s) = sn + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$$

$0,5dB\ ripple$, $\varepsilon = 0,3493114$, $\varepsilon^2 = 0,1220184$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	2,8627752					
2	1,5162026	1,4256245				
3	0,7156938	1,5348954	1,2529130			
4	0,3790506	1,0254553	1,7168662	1,1973856		
5	0,1789234	0,7525181	1,3095747	1,9373675	1,1424909	
6	0,0947626	0,4323669	1,1718613	1,5897635	2,1718446	1,1591761

$$H1(s) = \frac{2,8627752}{s + 2,8627752} \ ; H2(s) = \frac{1,5162026}{\sqrt{1,1220184}\,(s^2 + 1,4256245s + 1,5152026)}$$

Chebyshev filters

$$|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 + {T_n}^2(\Omega)}, Hn_{(s)} = \frac{K_n}{V_n(s)},$$

$$K_n = \begin{cases} \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & untuk\ n\ genap \\ b_0 & untuk\ n\ ganjil \end{cases}$$

$$V_n(s) = sn + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$$

$1\ dB\ ripple, \varepsilon = 0,5088471, \varepsilon^2 = 0,2589254$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	1,9652267					
2	1,1025103	1,0977343				
3	0,4913067	1,2384092	0,9883412			
4	0,2756276	0,7426194	1,4539248	0,9528114		
5	0,1228267	0,5805342	0,9743961	1,6888160	0,9368201	
6	0,0689069	0,3070808	0,9393461	1,2021409	1,9308256	0,9282510

$$H1(s) = \frac{1,9652267}{s + 1,9652267}\ ; H2(s) = \frac{1,1025103}{\sqrt{1,2589254}\,(s^2 + 1,0977343s + 1,1025103)}$$

Chebyshev filters

$$|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 + {T_n}^2\ (\Omega)}\ , Hn_{(S)} = \frac{K_n}{V_n(s)},$$

$$K_n = \begin{cases} \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & untuk\ n\ genap \\ b_0 & untuk\ n\ ganjil \end{cases}$$

$$V_n(s) = sn + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$$

$2\ dB\ ripple\ , \varepsilon = 0,7647831, \varepsilon^2 = 0,5848932$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	1,3075603					
2	0,6367681	0,8038164				
3	0,3268901	1,0221903	0,7378216			
4	0,2057651	0,5167981	1,2564819	0,7162150		
5	0,0817225	0,4593491	0,6934770	1,4995433	0,7064606	
6	0,0514413	0,2102706	0,7714618	0,8670149	1,7458587	0,7012257

$$H1(s) = \frac{1,3075603}{s + 1,3075603}\ ; H2(s) = \frac{0,6367681}{\sqrt{1,58448932}\ (s^2 + 0,8038164s + 0,6367681)}$$

EXAMPLE

- Rancang filter low pass chebyshev, bandwidth 1 rad/detik dengan spesifikasi sbb;
- Ripple di passband adalah 2dB, frekuensi cutoff 1 rad/detik
- Redaman pada stopband (diluar 1,3 rad/detik)≥20dB
- Jawab:
- $20\log|H_{n(j1)}| = 20log\sqrt{\frac{1}{1+\varepsilon^2}}$
- $20\log|H_{n(j1)}| = -2dB$
- $20\log|H_{n(j1,3)}| = 20log\sqrt{\frac{1}{A^2}}$
- $20\log|H_{n(j1,3)}| = -20dB$

EXAMPLE

- Diperoleh $\varepsilon = 0{,}76478$ dan $A = 10$
- $g = \sqrt{\frac{A^2-1}{\varepsilon^2}} = \sqrt{\frac{100-1}{0{,}76478^2}} = 13{,}01$
- $n = \left\lceil \frac{\log_{10}\left[g+\sqrt{g^2-1}\right]}{\log_{10}\left[\Omega_r+\sqrt{\Omega_r{}^2-1}\right]} \right\rceil = \left\lceil \frac{\log_{10}\left[13{,}01+\sqrt{13{,}01^2-1}\right]}{\log_{10}\left[1{,}3+\sqrt{1{,}3^2-1}\right]} \right\rceil = \lceil 4{,}3 \rceil = 5$
- $H_{5(s)} = \frac{K}{s^5+b_4 s^4=b_3 s^3+b_2 s^2+b_1 s+b_0}$
- $H_{5(s)} = \frac{0{,}08172}{s^5+0{,}7064s^4+1{,}4995s^3+0{,}6934s^2+0{,}4593s+{,}00817}$