SIGNALS AND SYSTEM

FEG2D3

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INTRODUCTION

- The concept and theory of signals and systems are needed in almost all electrical engineering fields and in many other engineering and scientific disciplines as well.
- In this chapter we introduce the mathematical description and representation of signals and systems and their classifications.
- We also define several important basic signals essential to our studies.

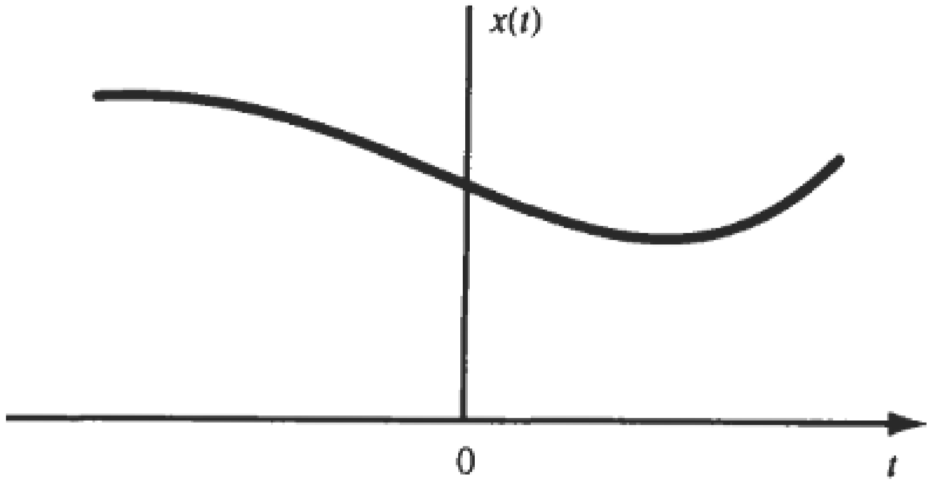
SIGNALS AND CLASSIFICATION OF SIGNALS

SIGNALS

- A signal is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon.
- For instance, in a RC circuit the signal may represent the voltage across the capacitor or the current flowing in the resistor.
- Mathematically, a signal is represented as a function of an independent variable `t`. Usually `t` represents time.
- Thus, a signal is denoted by **`x(t)`**

A. CONTINUOUS TIME SIGNALS

- x(t) is a continuous-time signal if t is a continuous variable.



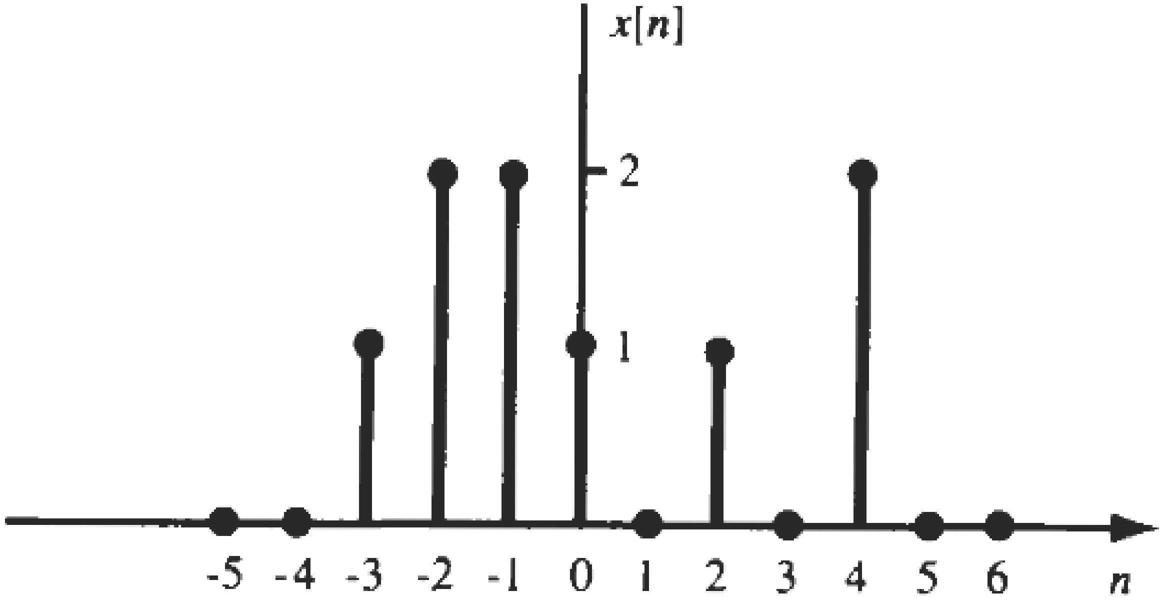
(a)

B. DISCRETE TIME SIGNALS

- $x(t)$ → if t is a discrete variable, $x(t)$ is defined at discrete times
- Then, $x(t)$ is a discrete-time signal
- Discrete time signals is defined at discrete times
- Often identified as a sequence of numbers
- Denoted by:

$$\{\mathbf{x}_n\} \text{ or } \mathbf{x}[\mathbf{n}], \text{ where } n = \text{integer}$$

B. DISCRETE TIME SIGNALS



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B. DISCRETE TIME SIGNALS

- $x[n]$ may represent a phenomenon for which the independent variable is inherently discrete
- $x[n]$ may be obtained by sampling a continuous time signal $x(t)$ such as:

$$x(t_0),\ x(t_1),\ \ldots,\ x(t_n),\ \ldots$$

Or in a shorter form as

$$x[0],\ x[1],\ \ldots,\ x[n],\ \ldots$$

Or

$$x_0, x_1,\ x_2,\ \ldots,\ x_n,\ \ldots$$

So…

$$x_n = x[n] = x(t_n)$$

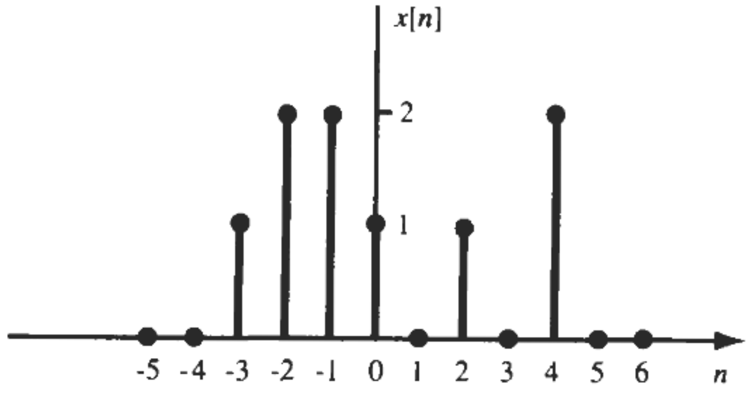
B. DISCRETE TIME SIGNALS

- x_n's are called samples and the time interval between them is called the sampling interval.
- When the sampling intervals are equal / uniform sampling, then

$$x_n = x[n] = x(nT_s),$$

where the constant T, is the sampling interval

B. DISCRETE TIME SIGNALS



(b)

A discrete-time signal x[n] can be defined in two ways:

1. We can specify a rule for calculating the nth value of the sequence. For example:

$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$\{x_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \left(\frac{1}{2}\right)^n, \ldots\right\}$$

2. We can also explicitly list the values of the sequence. For example, the sequence shown in (b) can be written as

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2, 0, 0, \ldots\}$$

$$\{x_n\} = \{1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2\}$$

B. DISCRETE TIME SIGNALS

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2, 0, 0, \ldots\}$$

$$\{x_n\} = \{1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2\}$$

- We use the arrow to denote the $n = 0$ term.
- We shall use the convention that if no arrow is indicated, then the first term corresponds to $n = 0$ and all the values of the sequence are zero for $n < 0$.
- The sum and product of two sequences are defined as follows

$$\{c_n\} = \{a_n\} + \{b_n\} \longrightarrow c_n = a_n + b_n$$

$$\{c_n\} = \{a_n\}\{b_n\} \longrightarrow c_n = a_n b_n$$

$$\{c_n\} = \alpha\{a_n\} \longrightarrow c_n = \alpha a_n \qquad \alpha = \text{constant}$$

C. ANALOG AND DIGITAL SIGNALS

- If a continuous-time signal $x(t)$ can take on any value in the continuous interval (a, b), where a may be $-\infty$ *and b may be* $+\infty$*, then the continuous-time signal* $x(t)$ *is called an* **analog signal.**
- If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a **digital signal.**

D. REAL AND COMPLEX SIGNALS

- A signal x(t) is a real signal if its value is a real number, and a signal x(t) is a complex signal if its value is a complex number.
- A general complex signal x(t) is a function of the form

$$x(t) = x_1(t) + jx_2(t)$$

Where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$

* Note that t represents either a continuous or a discrete variable.

E. DETERMINISTIC AND RANDOM SIGNALS

- Deterministic signals are those signals whose values are completely specified for any given time.
- Thus, a deterministic signal can be modeled by a known function of time t.
- Random signals are those signals that take random values at any given time and must be characterized statistically.
- Random signals will not be discussed in this text.

F. EVEN AND ODD SIGNALS

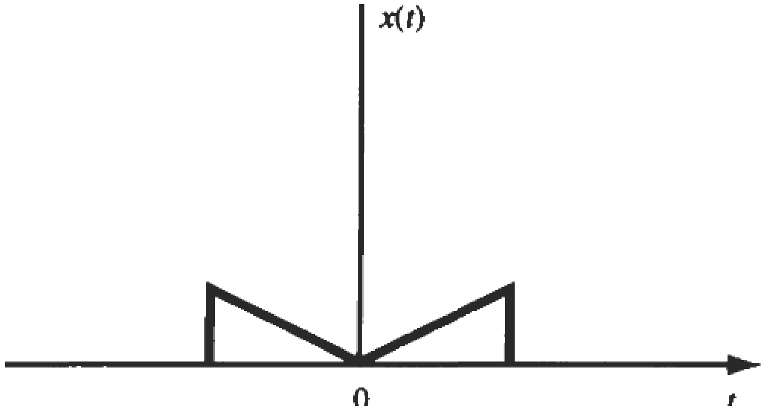
- A signal x(t) or x[n] is referred to as **an even signal** if:

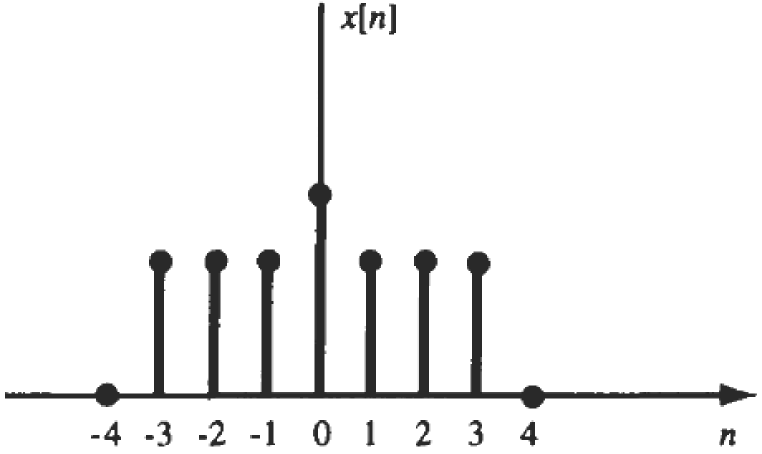
$$x(-t) = x(t)$$
$$x[-n] = x[n]$$

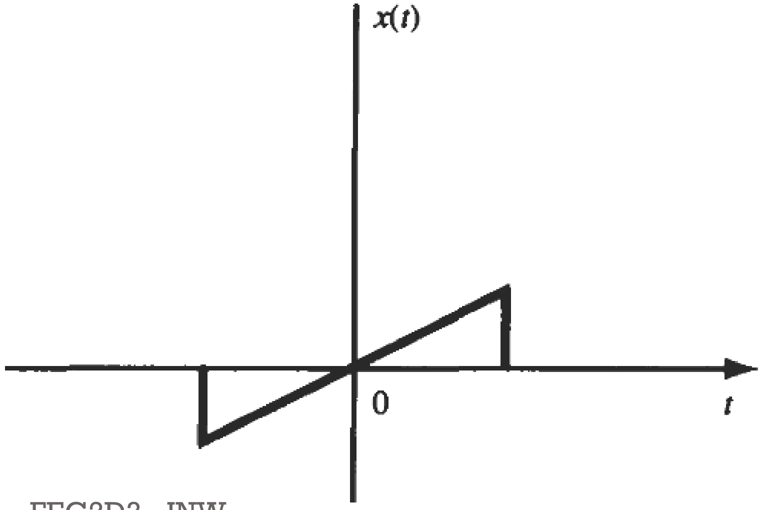
- A signal x(t) or x[n] is referred to as **an odd signal** if:

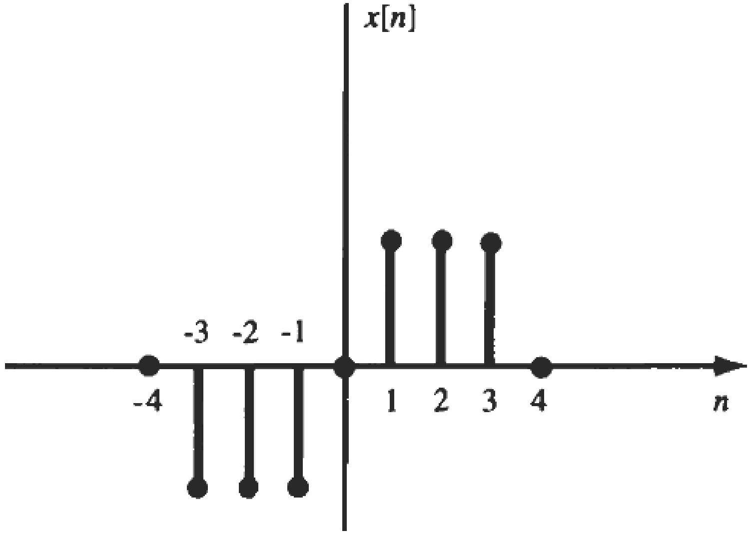
$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

F. EVEN AND ODD SIGNALS









F. EVEN AND ODD SIGNALS

- Any signal x(t) or x[n] can be expressed as a sum of two signals, one of which is even and one of which is odd.

$$x(t) = x_e(t) + x_0(t)$$
$$x[n] = x_e[n] + x_0[n]$$

Where:

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$ even part of x(t)

$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$$ even part of x[n]

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$ odd part of x(t)

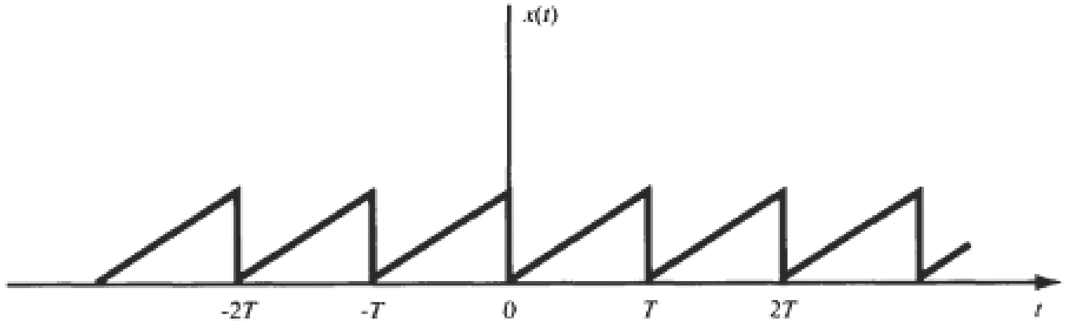
$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$$ odd part of x[n]

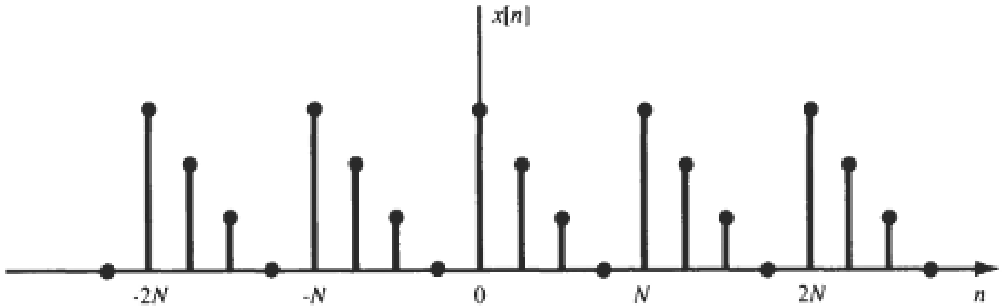
- the product of two even signals or of two odd signals is an **even signal**
- that the product of an even signal and an odd signal is an **odd signal**

E. PERIODIC AND NON PERIODIC SIGNALS

- A continuous-time signal x(t) is said to be periodic with period T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \text{ all } t$$





$$x(t+mT) = x(t) \text{ all } t \text{ and any integer } m$$

H. ENERGY & POWER SIGNALS

- Consider `v(t)` to be the voltage across a resistor `R` producing a current `i(t)`. The instantaneous power `p(t)` per ohm is defined as:

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

- Total energy E and average power P on a per-ohm basis are:

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules}$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts}$$

H. ENERGY & POWER SIGNALS

- For an arbitrary continuous-time signal x(t), the normalized energy content E of x(t) and x[n] are defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- Normalized average power P of x(t) and x[n] are defined as:

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \qquad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

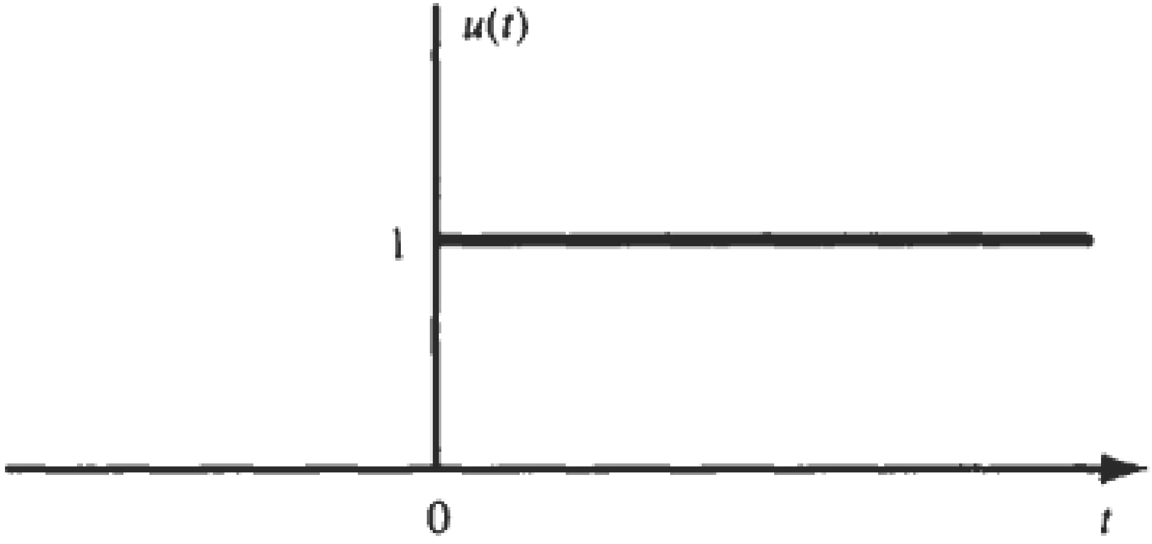
BASIC CONTINUOUS TIME SIGNALS

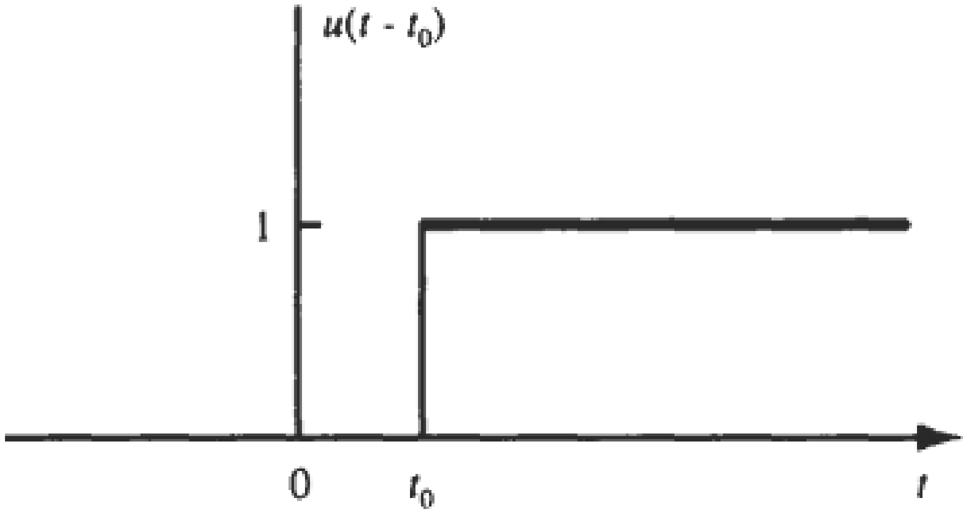
A. UNIT STEP FUNCTION

- The unit step function u(t), also known as the Heaviside unit function, is defined as:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$



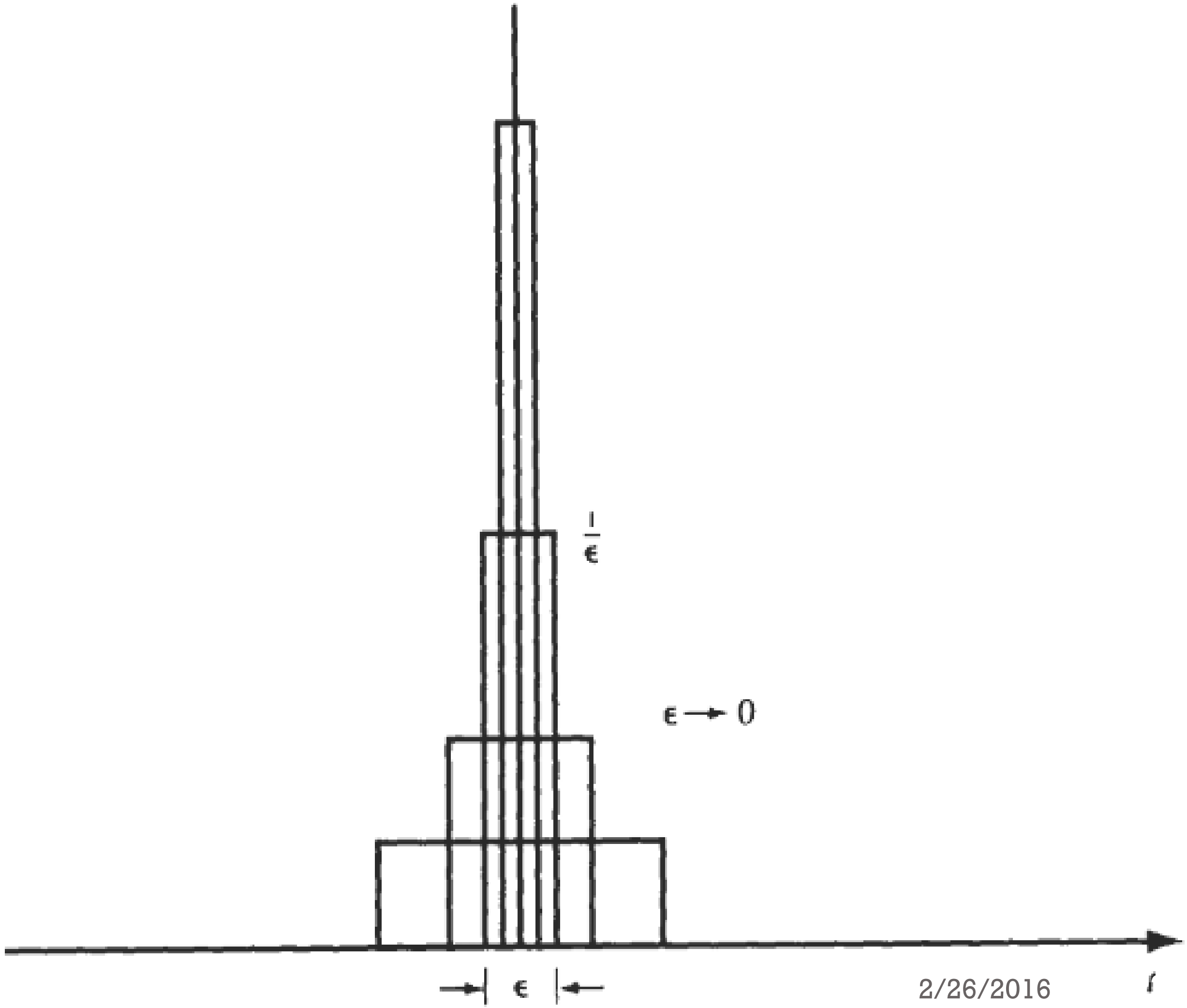


B. UNIT IMPULSE FUNCTION/IMPULS SIGNAL

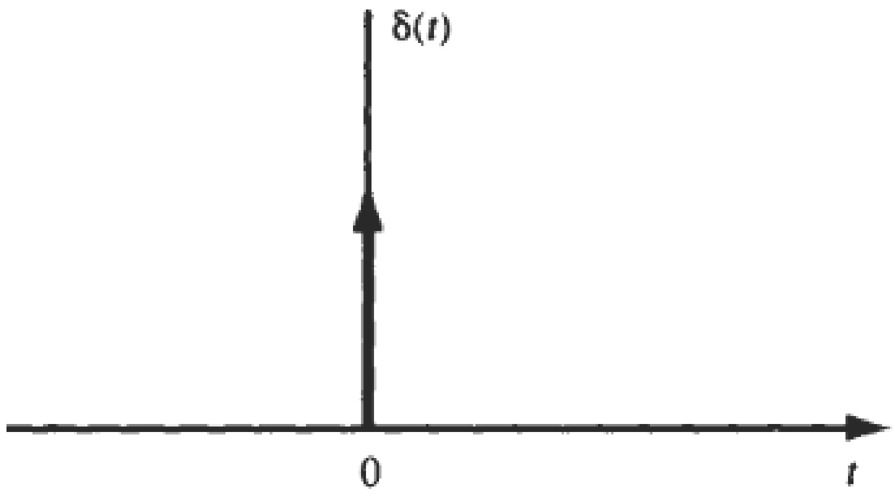
- The unit impulse function $\delta(t)$, also known as the Dirac delta function, plays a central role in system analysis.
- Traditionally, $\delta(t)$ is often defined as the limit of a suitably chosen conventional function having unity area over an infinitesimal time interval as shown below:

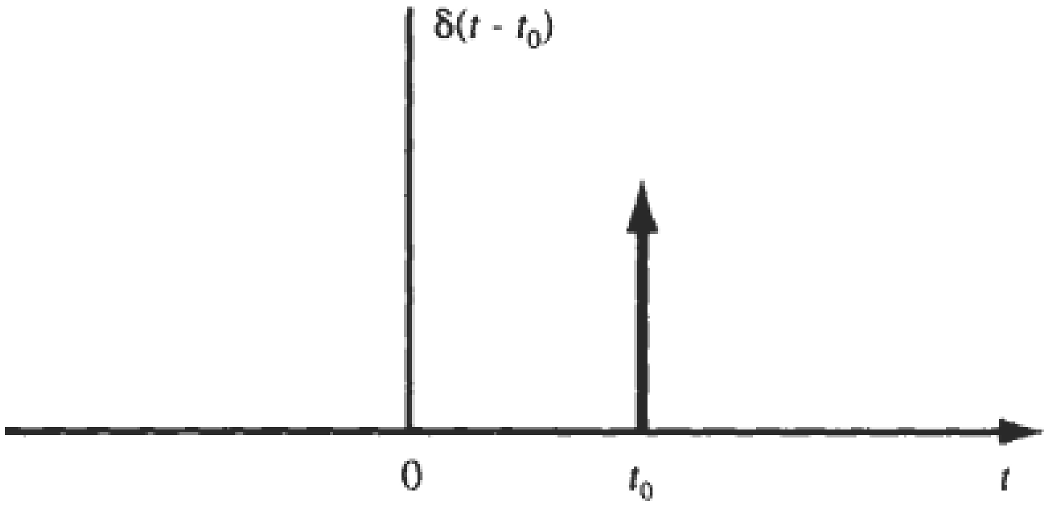
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(t)\, dt = 1$$



B. UNIT IMPULSE FUNCTION/IMPULS SIGNAL





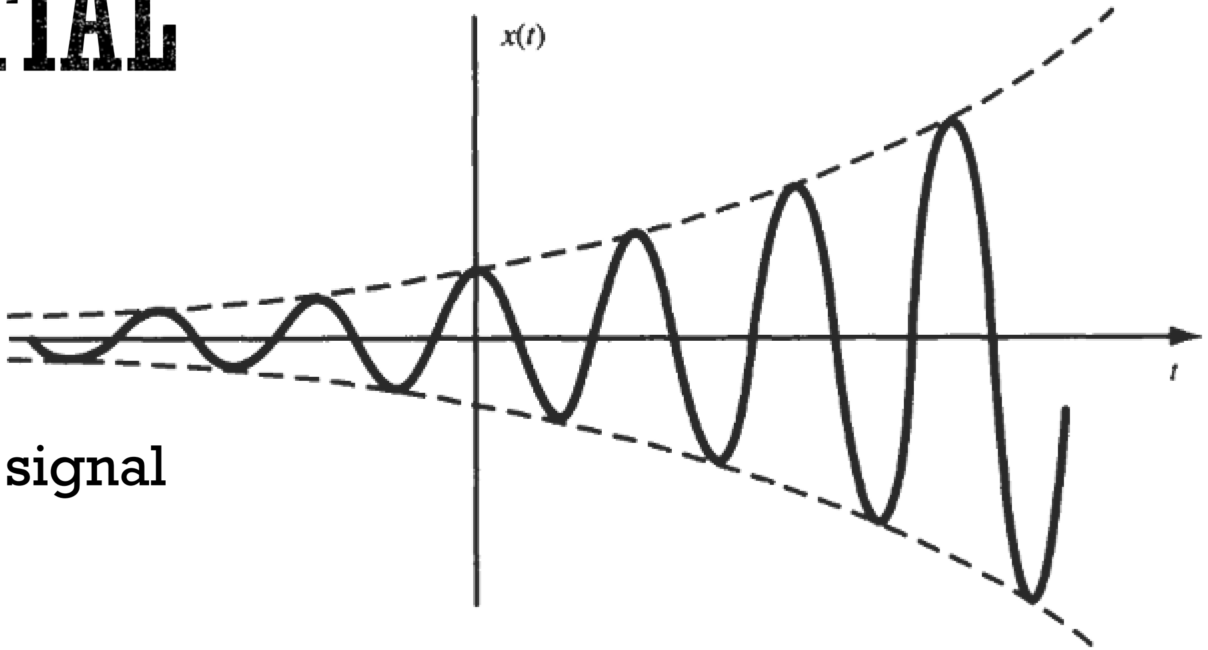
C. COMPLEX EXPONENTIAL SIGNALS

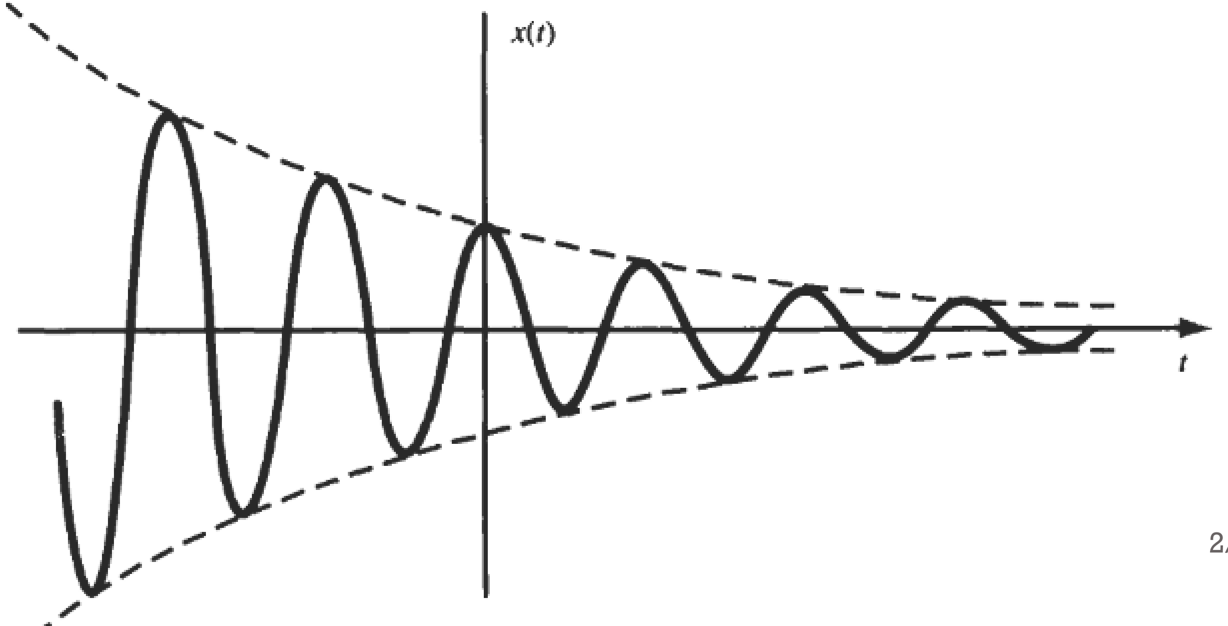
- The complex exponential signal

  $x(t) = e^{j\omega 0t}$

  is an important example of a complex signal

1. Exponentially Increasing sinusoidal signal
2. Exponentially decreasing sinusoidal signal





C. COMPLEX EXPONENTIAL SIGNALS

- By using Euler's formula……

$$x(t) = e^{j\omega 0t} = cos\omega_0 t + jsin\omega_0 t$$

- X(t) is a complex signals, whose real part is $cos\omega_0 t$ and imaginary part is $jsin\omega_0 t$
- The fundamental period T_0 of x(t) is given by:

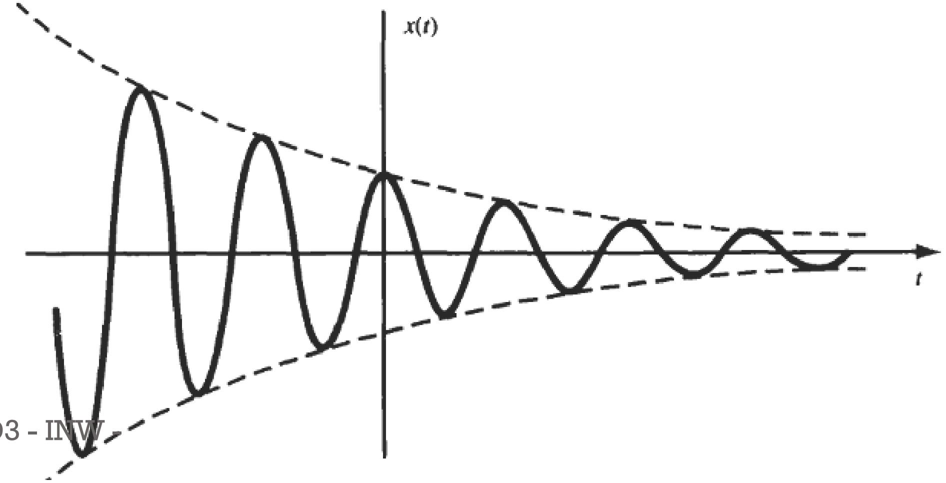
$$T_0 = \frac{2\pi}{\omega_0}$$

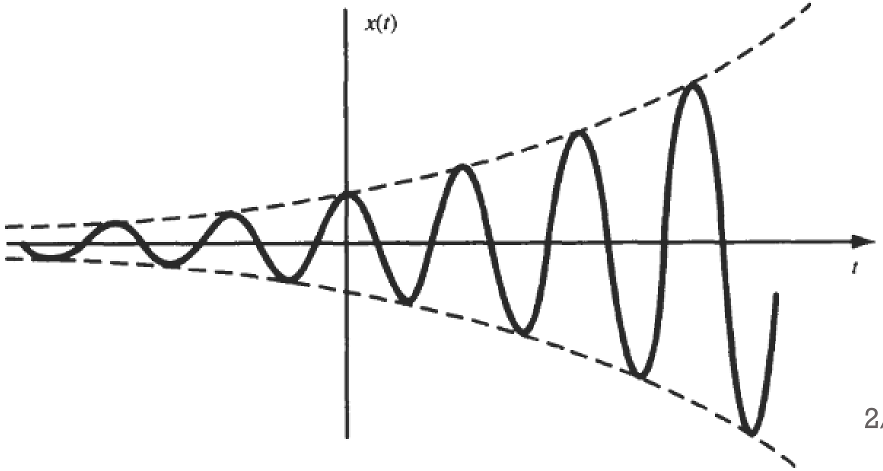
GENERAL COMPLEX EXPONENTIAL SIGNALS

- Lets $s = \sigma + j\omega$ be a complex number. We define $x(t)$ as:

$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(\cos \omega t + jsin\omega t)$$

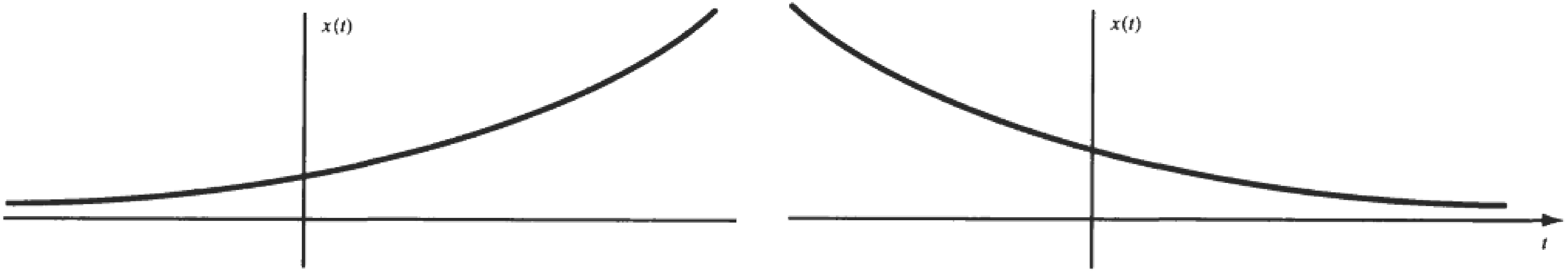
- Then signal x(t) is known as a general complex exponential signal whose real part $e^{\sigma t}cos\omega t$ and imaginary part $e^{\sigma t}sin\omega t$ are exponentially increasing (σ> 0) or decreasing (σ< 0) sinusoidal signals





REAL EXPONENTIAL SIGNALS

- Note that if $s = \sigma$ (a real number) *reduce a real exponential signal



Continuous-time real exponential signals
$\sigma > 0$ & $\sigma < 0$ (growing exp & decaying exp)

D. SINUSOIDAL SIGNAL

- A continuous-time sinusoidal signal can be expressed as

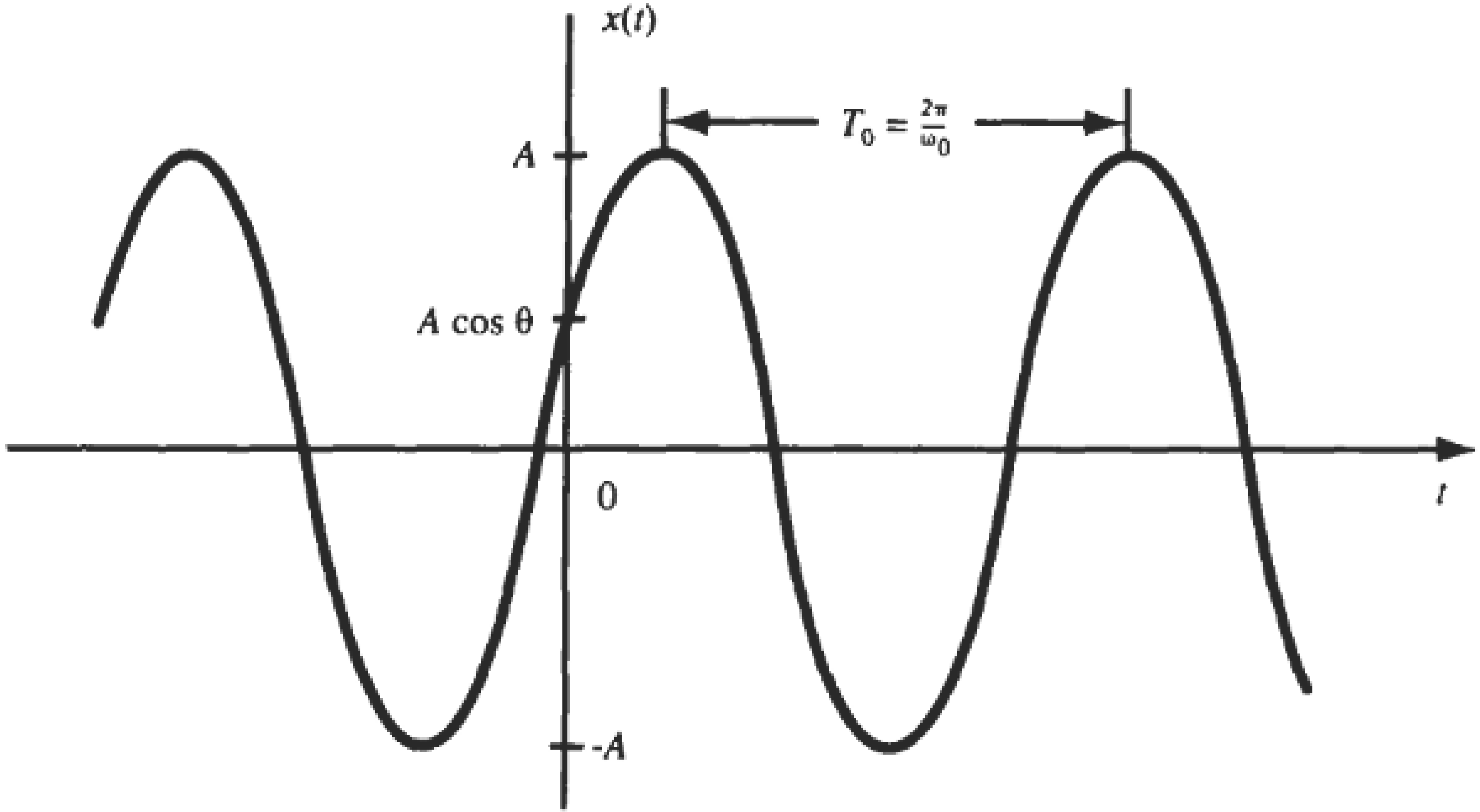
$$x(t) = Acos(\omega_0 t + \theta)$$

  - *A is the amplitude (real),*
  - *ω is the radian frequency in radians per second, and*
  - *θ is* the phase angle in radians

- Period: $T_0 = \frac{2\pi}{\omega_0}$

- Fundamental frequency: $f_0 = \frac{1}{T_0} \rightarrow \omega_0 = 2\pi f_0$ fundamental angular frequency

D. SINUSOIDAL SIGNAL

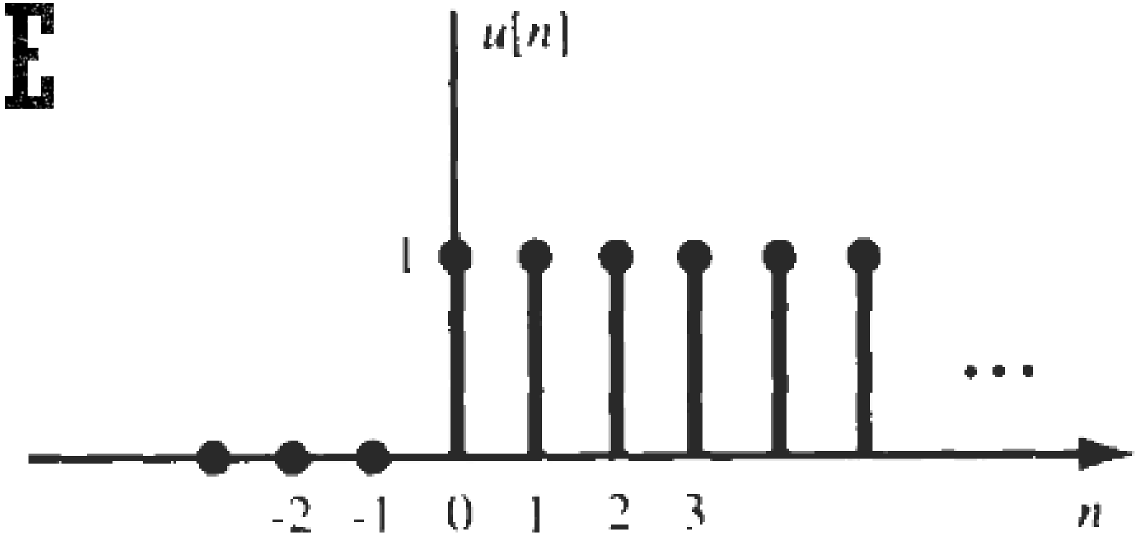


BASIC DISCRETE TIME SIGNALS

A. UNIT STEP SEQUENCE

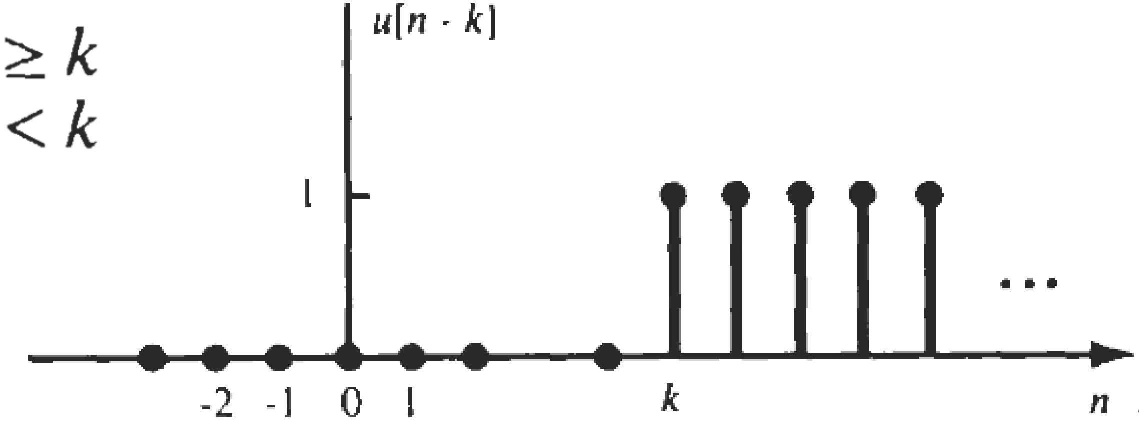
- The unit step sequence u[n] is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



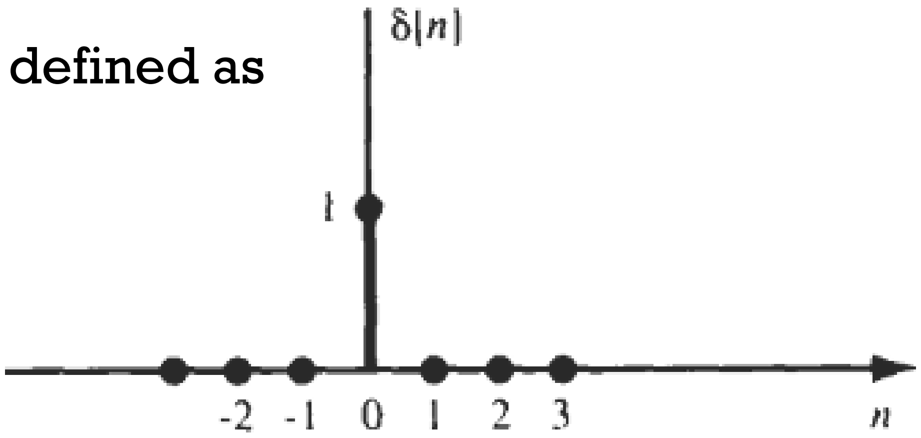
- Note that the value of u[n] at n = 0 is defined [unlike the continuous-time step function u(f) at t = 01 and equals unity

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases}$$



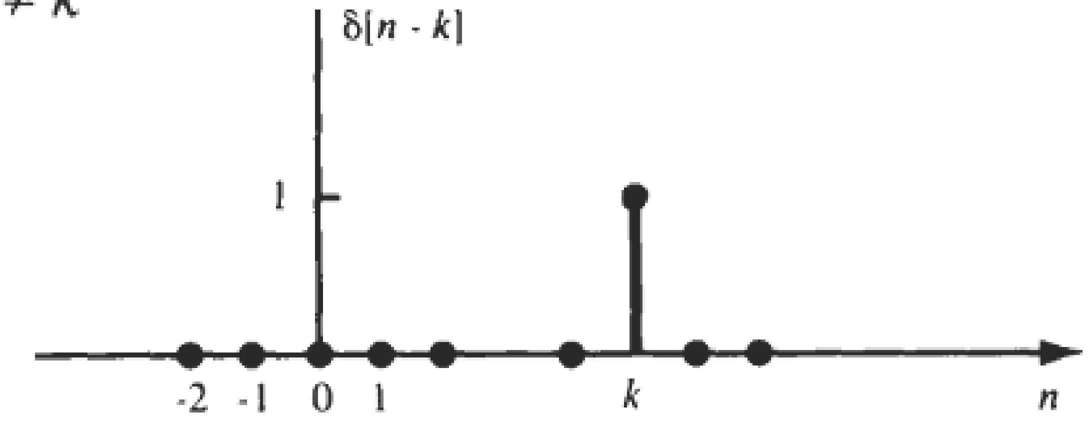
B. THE UNIT IMPULSE SEQUENCE

- The unit impulse (or unit sample) sequence $\delta[n]$ is defined as



$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

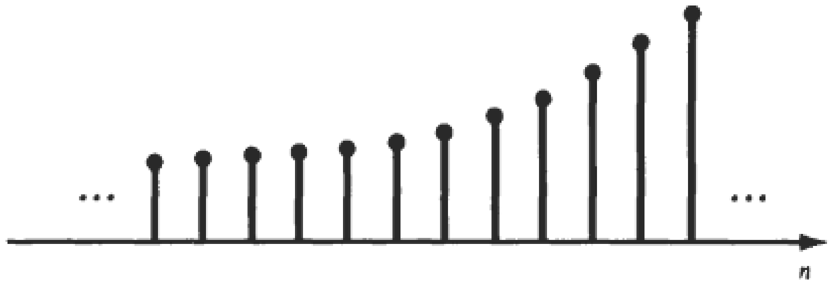
$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$



C. COMPLEX EXPONENTIAL SEQUENCES

- The complex exponential sequence is of the form:

$$x[n] = e^{\Omega_0 n} = \cos \Omega_0 n + j sin \Omega_0 n$$



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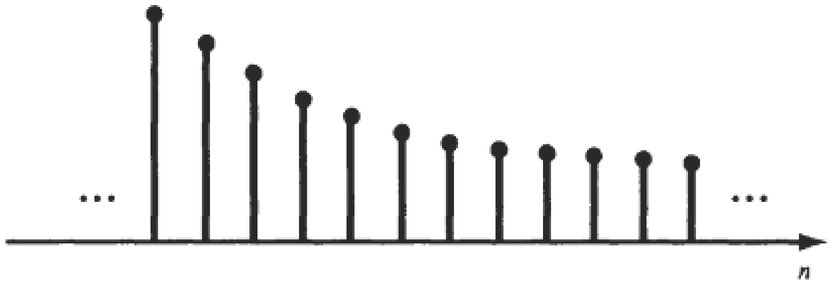

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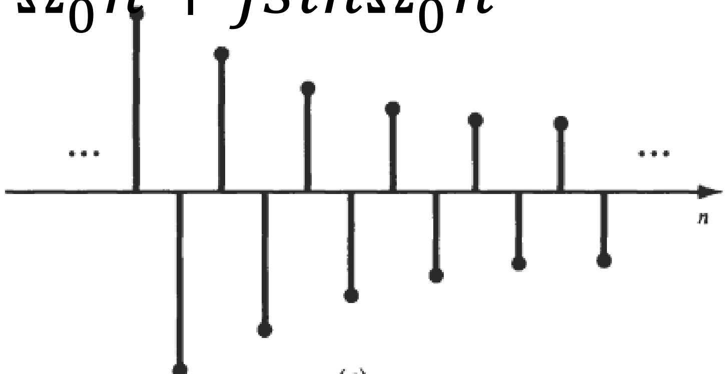

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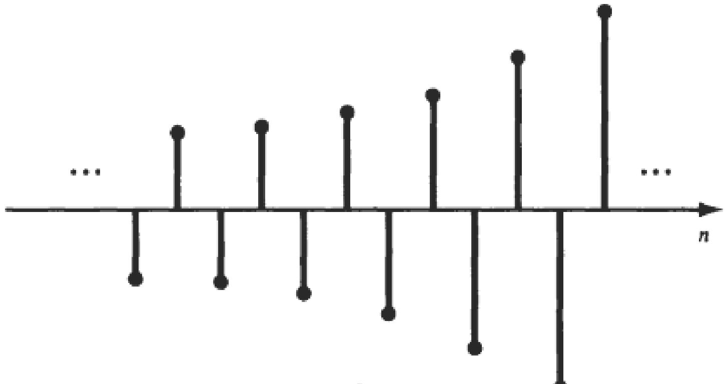
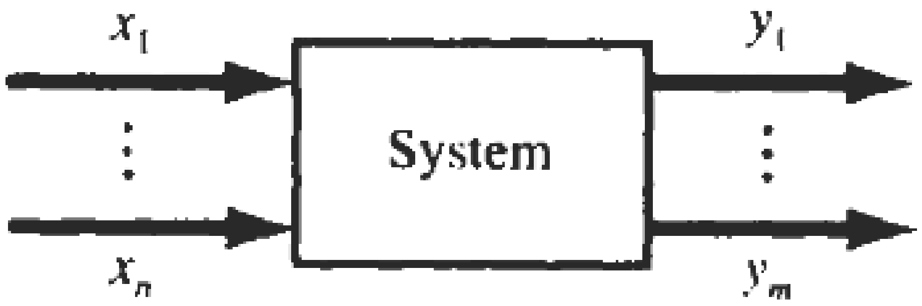


(d)

SYSTEM AND CLASSIFICATION OF SYSTEMS

A. SYSTEM REPRESENTATION

- A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.
- Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y.
- This transformation is represented by the mathematical notation $y = Tx$
- Where T is the operator representing some well-defined rule by which x is transformed into y.



B. CONTINUOUS TIME AND DISCRETE TIME SYSTEMS

- If the input and output signals x and y are continuous-time signals, then the system is called a continuous-time system

- If the input and output signals are discrete-time signals or sequences, then the system is called a discrete-time system

C. SYSTEM WITH MEMORY AND WITHOUT MEMORY

- A system is said to be memoryless **if the output at any time depends on only the input at that same time.**
- Otherwise, the system is said to have memory.
- An example of a memoryless system is a resistor R with the input x(t) taken as the current and the voltage taken as the output y(t). The input-output relationship (Ohm's law) of a resistor is:

$$y(t) = Rx(t)$$

- An example of a system with memory is a capacitor *C with the current as the input x(t)* and the voltage as the output *y(t)*

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(T)dT$$

- A second example of a system with memory is a discrete-time system whose input and output sequences are related by

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

D. CAUSAL AND NON CAUSAL

- A system is called causal if its output y(t) at an arbitrary time t = t, depends on only the input x(t) for t ≤ to.
- That is, the output of a causal system at the present time depends on only **the present and/or past values of the input, not on its future values.**
- Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system.
- **A system is called *noncausal if it is not causal.***

$$y(t) = x(t+1)$$
$$y[n] = x[-n]$$

**Note that all memoryless systems are causal, but not vice versa.

E. LINIER SYSTEMS AND NONLINIER SYSTEMS

- notation $y = Tx$
- If the operator T satisfies the following two conditions, then T is called a linear operator and the system represented by a linear operator T is called **a linear system:**

1. Additivity

   Given that $Tx_1 = y_1$ and $Tx_2 = y_2$ then $T\{x_1+x_2\} = y_1 + y_2$

2. Homogeneity or scaling

   $$T\{\alpha x\} = \alpha y$$

- Any system that does not satisfy Eq. 1 and/or Eq. 2 is classified as a **nonlinear system.**
- Example non linier system: $y = x^2$ and $y = cos\ x$

F. TIME-INVARIANT AND TIME VARYING SYSTEM

- A system is called **time-invariant** if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, for a continuous-time system, the system is time-invariant if:

$$T\{x(t-\tau)\} = y(t-\tau)$$

$$T\{x[n-k]\} = y[n-k]$$

- A system which does not satisfy those equation (continuous-time system) or (discrete-time system) is called **a time-varying system.**

H. LINIAR TIME INVARIANT SYSTEMS

- If the system is linear and also time-invariant, then it is called a linear rime-invariant (LTI) system.

H. STABLE SYSTEMS

- A system is bounded-input/bounded-output (BIBO) stable if for any bounded input x defined by:

$$|x| \leq k_1$$

- the corresponding output y is also bounded defined by

$$|y| \leq k_2$$

- Where k, and k, are finite real constants.

I. FEEDBACK SYSTEM

- A special class of systems of great importance consists of systems having feedback.
- In a feedback system, the output signal is fed back and added to the input to the system as shown